2019

STATISTICS

(Honours)

Paper : STA-HC-1026

(Calculus)

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Choose the correct option for each of the following : 1×7=7

(a) According to L' Hospital's rule for indeterminate form $\frac{\infty}{\infty}$, under certain conditions imposed upon the functions $f(x)$ and $g(x)$, if $\lim_{x \to a} \frac{f'(x)}{g'(x)} = l$, then the value of $\lim_{x \to a} \frac{f(x)}{g(x)}$ is

(i) l

(ii) $\frac{1}{l}$

(iii) 0

(b) The gamma function $\int_0^\infty x^{m-1}e^{-x}\,dx$ converges for

(i) $m > 1$

(ii) $m > 0$

(iii) all real values of m

(c) The order and degree of the differential equation

$$\frac{x + y\frac{dy}{dx}}{x\frac{dy}{dx} - y} = \sqrt{\frac{1 - x^2 - y^2}{x^2 + y^2}}$$

is

(i) 1, 1

(ii) 1, 2

(iii) 2, 1

(d) Extreme value of a function exists if and only if the first non-zero derivative of the function is of ____ order.

(i) odd

(ii) even

(e) The 'integrating factor' (in context of obtaining solution of a differential equation) of the differential equation

$$\cos x\frac{dy}{dx} + y\sin x = 1$$

is

(i) $\log(\sec x)$ (ii) $\tan x$ (iii) $\sec x$

(f) Which of the following is not a linear partial differential equation?

(i) $\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = z + xy$

(ii) $\frac{\partial^2 z}{\partial x^2} = \left(1 + \frac{\partial z}{\partial y}\right)^{1/2}$

(iii) $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = xyz$

(g) The general solution of the linear differential equation

$$\frac{d^2y}{dx^2} + P_1\frac{dy}{dx} + P_2 y = 0$$

when the auxiliary equation has imaginary roots is of the form

(i) $y = ce^{\alpha x}\cos(\beta x + \varepsilon)$

(ii) $y = c_1 e^{m_1 x} + c_2 e^{m_2 x}$

(iii) $y = (c_1 + c_2 x)e^{\alpha x}$

Here, the symbols have their usual meanings.

2. Answer the following questions : 2×4=8

(a) If $x = r\cos\theta$ and $y = r\sin\theta$, then find the value of $\frac{\partial^2\theta}{\partial x \partial y}$.

(b) Form the partial differential equation by eliminating the arbitrary constants a and b from

$$(x-a)^2 + (y-b)^2 + z^2 = \lambda^2$$

(c) Show that

$$\Gamma\left(\frac{7}{2}\right) = \frac{15}{8}\sqrt{\pi}$$

(d) Solve :

$$\left(\frac{y^2 z}{x}\right) p + xzq = y^2$$

3. Answer any *three* from the following questions : 5×3=15

(a) Find the point of maxima and the minima of the function $x^3 - 12x^2 + 45x$ in the interval [0, 7].

(b) Solve :

$$(x\cos x)\frac{dy}{dx} + y(x\sin x + \cos x) = 1$$

(c) Use the relation between gamma and beta function to show that

$$\int_0^1 x^{\frac{3}{2}}(1-x)^{\frac{3}{2}}\,dx = \frac{3\pi}{128}$$

(d) Evaluate the limit :

$$\lim_{x\to 0} \frac{(1+x)^{\frac{1}{x}} - e}{x}$$

(e) Solve the differential equation

$$\left(\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4\right)y = x^2$$

when $x = 0$, $y = \frac{3}{8}$ and $\frac{dy}{dx} \cdot y = 1$.

Answer the following questions : 10×3=30

4. *(a)* *(i)* Evaluate : 6

$$\frac{1}{\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2} e^{e^x}$$

(ii) Solve : 4

$$\frac{d^2y}{dx^2} = \cos nx$$

Or

(b) Define 'Jacobian'. If $u = \frac{x^2 + y^2 + z^2}{x}$,

$v = \frac{x^2 + y^2 + z^2}{y}$ and $w = \frac{x^2 + y^2 + z^2}{z}$,

then find $\frac{\partial(x, y, z)}{\partial(u, v, w)}$. 3+7=10

5. (a) Find the maximum and minimum values of $x^2 + y^2 + z^2$ subject to the conditions

$$\frac{x^2}{4} + \frac{y^2}{5} + \frac{z^2}{25} = 1 \text{ and } z = x + y$$ 10

Or

(b) (i) Solve the Clairaut's equation : 5

$$y = px + p - p^2, \text{ where } p = \frac{dy}{dx}$$

(ii) Show that the differential equation

$$xdx + ydy = \frac{a^2(xdy - ydx)}{x^2 + y^2}$$

is exact and hence solve it. 5

6. (a) (i) Evaluate the integral : 5

$$\iint_R (x^2 + 2y)\,dxdy, \; R = [0, 1; 0, 2]$$

(ii) Prove that a necessary and sufficient condition that the differential equation $Mdx + Ndy = 0$ be exact is that

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$ 5

Or

(b) (i) Find the first- and second-order partial derivatives of $z = x^3 + y^3 - 3axy$ and verify that

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial^2 z}{\partial y \partial x}$$ 2+3=5

(ii) Given $\int_0^{\infty} \frac{x^{p-1}}{1+x}\,dx = \frac{\pi}{\sin p\pi}$, show that

$$\Gamma(p)\Gamma(1-p) = \frac{\pi}{\sin p\pi}, \text{ when } 0 < p < 1$$ 5

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