2019

STATISTICS

(Major)

Paper : 5.1

(Sampling Distribution and Statistical Inference–I)

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Answer the following questions as directed : $1 \times 7 = 7$

(a) The points of inflexion of *F*-distribution always exist.

(State True or False)

(b) State Cramer-Rao lower bound of the variance of an unbiased estimator.

(c) The degrees of freedom (df) of a chi-square statistic is 3, what will be the df of the corresponding Fisher's t-statistic?

(d) What is asymptotic unbiasedness?

(e) State the cumulative distribution function (c.d.f.) of the smallest order statistic $x_{(1)}$.

(f) Generally the method of moments yields less efficient estimators than those obtained from the principle of _____.

(Fill in the blank)

(g) Define 'linear orthogonal transformation'.

2. Answer the following questions : 2×4=8

(a) Under what conditions, is χ^2 (chi-square) test valid?

(b) For large n, prove with usual notation

$$S.E(s^2) = \sigma^2 \times \sqrt{\frac{2}{n}}$$

(c) Find the MLE of θ in

$$f(x, \theta) = (1+\theta)x^{\theta}, \quad 0 < x < 1$$

based on an independent sample of size n.

(d) A random sample of size 4 is drawn from the discrete uniform distribution

$$P(X = x) = \frac{1}{6}, \quad x = 1, 2, \cdots, 6$$

Obtain the distribution function of the largest statistic.

3. Answer any *three* parts : 5×3=15

(a) Let $\hat{\theta}_n$ be an unbiased estimator of θ_n and $\text{var}(\hat{\theta}_n) = \sigma_n^2$. Also assume that

$$\left.\begin{aligned} \theta_n &\to \theta \\ \text{and } \sigma_n &\to 0 \end{aligned}\right\} \text{ as } n \to \infty$$

Then prove that $\hat{\theta}_n$ is consistent estimator of θ.

(b) Show that the m.g.f. of $Y = \log \chi^2$, where χ^2 follows chi-square distribution with n d.f. is

$$M_Y(t) = 2^t \Gamma(\tfrac{n}{2}+t) \big/ \Gamma(n/2)$$

(c) Write down the probability function of r^2 where r is the sample correlation coefficient. If X is an F variate with 2 and $n(n \geq 2)$ degrees of freedom, then show that

$$P(X \geq K) = \left(1 + \frac{2K}{n}\right)^{-n/2}$$

(d) Let T_1 be the MVUE of θ and T_2 be another unbiased estimator of θ, then prove that the linear combination of T_1 and T_2 will not be MVUE of θ.

(e) Show that for t-distribution with n d.f. the mean deviation about mean is

$$\sqrt{n}\,\Gamma\left(\tfrac{n-1}{2}\right) / \Gamma\left(\tfrac{1}{2}\right)\Gamma\left(\tfrac{n}{2}\right)$$

4. Answer either *(a)* or *(b)* of the following questions :

(a) (i) Describe briefly the method of minimum chi-square. 3

(ii) Derive Snedecor's F-distribution. 7

(b) (i) Find the p.d.f. of the rth order statistic $X_{(r)}$ in a random sample of size n from the exponential distribution

$$f(x) = \alpha e^{-\alpha x},\ \alpha > 0,\ x \geq 0$$ 4

(ii) Show that $X_{(r)}$ and $W_{rs} = X_{(s)} - X_{(r)}$, are independently distributed. 4

(iii) What is the distribution of $W_1 = X_{(r+1)} - X_{(r)}$? 2

5. Answer either *(a)* or *(b)* of the following questions :

(a) (i) Write a brief note on the method of moments for estimating parameters. 3

(ii) A random variable X takes the values 0, 1, 2 with respective probabilities

$$\frac{\theta}{4N} + \frac{1}{2}\left(1 - \frac{\theta}{N}\right),\ \frac{\theta}{2N} + \frac{\alpha}{2}\left(1 - \frac{\theta}{N}\right)$$

$$\text{and } \frac{\theta}{4N} + \frac{1-\alpha}{2}\left(1 - \frac{\theta}{N}\right)$$

where N is a known number and α and θ are unknown parameters. If 75 independent observations on X yielded the values 0, 1, 2 with frequencies 27, 38, 10 respectively, estimate the parameters θ and α by the method of moments. 7

(b) *(i)* Let $x_1, x_2, \cdots, x_n$ be a random sample of n observations from Bernoulli population with parameter θ. Find the estimator of θ by the method of minimum chi-square. 4

(ii) If X_1 and X_2 are two independent random variables having common density function

$$f(x) = e^{-x}, \quad 0 \le x < \infty$$

show that $u = \frac{X_1}{X_2}$ has F-distribution with (2, 2) d.f. 6

6. Answer either *(a)* or *(b)* of the following questions :

(a) *(i)* State important applications of F-distribution. 3

(ii) Let the estimator of θ in $f(x, \theta)$ be T, where T is a sufficient statistic. If the MLE of θ exists, then show that it is the function of the sufficient statistic T. 7

(b) *(i)* State the important properties of MLE. 3

(ii) For 2×2 contingency table

a	b
c	d

prove that chi-square statistics for testing independence of attributes is given by

$$\chi^2 = \frac{N(ad - bc)^2}{(a+b)(c+d)(a+c)(b+d)}$$

where $N = a + b + c + d$. 7

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