(ii) Show that for t-distribution with n d.f. mean deviation about mean is given by

$$\frac{\sqrt{n}\,\Gamma\left(\frac{n-1}{2}\right)}{\sqrt{\pi}\,\Gamma\left(\frac{n}{2}\right)}$$ 5

6. (a) (i) Derive the expression for the standard error of the mean of a random sample of size n and sample proportion. 5

(ii) Write down some familiar applications of order statistics. 5

Or

(b) (i) If $n_2 \to \infty$ in $F(n, n_2)$ distribution, then show that $\chi^2 = n_1 F$ follows Chi-square distribution with n_1 d.f. 7

(ii) Explain how the student's t-distribution is used to test the difference between the means of two samples which are paired together. 3

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3 (Sem-3/CBCS) STA HC 1

2021

(Held in 2022)

STATISTICS

(Honours)

Paper : STA-HC-3016

(Sampling Distributions)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions as directed : $1 \times 7 = 7$

(a) For random sample of size 2 drawn from $N(0, \sigma^2)$ population, the expected value of the smallest order statistic is

(i) $-\frac{\sigma^2}{\sqrt{\pi}}$

(ii) $-\frac{\sigma}{\sqrt{\pi}}$

(iii) $\dfrac{2\sigma^2}{\sqrt{\pi}}$

(iv) None of the above

(Choose the correct option)

(b) If X and Y are two independent Chi-square variates with n_1 and n_2 degrees of freedom respectively, then $u = \dfrac{X}{Y}$ follows

(i) $\beta_2\left(\dfrac{n_1}{2}, \dfrac{n_2}{2}\right)$

(ii) $\beta_1(n_1, n_2)$

(iii) F-distribution

(iv) None of the above

(Choose the correct option)

(c) If X is distributed as a Chi-square variate with n d.f., then for large N, $\sqrt{2x}$ is distributed as

(i) $N(2n, 1)$

(ii) $N\left(\sqrt{2n}, 1\right)$

(iii) $N\left(\sqrt{2n}, n\right)$

(iv) None of the above

(Choose the correct option)

(d) For testing the hypothesis 'population correlation ratio is zero'. The test statistics is

(i) $\dfrac{\eta^2}{1-\eta^2} \cdot \dfrac{N-h}{h-1}$

(ii) $\dfrac{1-\eta^2}{\eta^2} \cdot \dfrac{N-h-1}{h}$

(iii) $\dfrac{\eta^2}{1-\eta^2} \cdot \dfrac{N-h-1}{h^2}$

(iv) None of the above

(Choose the correct option)

(e) If a statistic t follows students t-distribution with n d.f., then t^2 follows ______. *(Fill in the blank)*

(f) 95% confidence limits for population proportion are

(i) $p \pm 1.96\sqrt{\frac{pa}{n}}$

(ii) $p \pm 2.58\sqrt{\frac{pq}{n}}$

(iii) $p \pm 2.33\sqrt{\frac{pq}{n}}$

(iv) None of the above

(Choose the correct option)

(g) The moment generating function of t-distribution does not exist.

*(State True **or** False)*

2. Answer the following questions : 2×4=8

(a) Explain the terms 'level of significance' and 'critical region'.

(b) Obtain cumulant generating function of Chi-square distribution. Hence obtain mean and variance.

(c) A random sample of size 4 is drawn from the discrete uniform distribution

$$P(X=x)=\frac{1}{6}\ ;\ x=1, 2, 3, 4, 5, 6$$

Obtain the distribution of the smallest and largest order statistic.

(d) Let $X_1, X_2, ..., X_n$ be a random sample from $N(0, 1)$. Let us further define $\bar{x}_k = \frac{1}{k}\sum_{i=1}^{k} X_i$ and

$$\bar{X}_{n-k} = \frac{1}{n-k}\sum_{i=k+1}^{n} X_i$$

Find the distribution of

$$k\bar{X}_k^{-2} + (n-k)\bar{X}_{n-k}^{-2}.$$

3. Answer **any three** of the following questions : 5×3=15

(a) Derive the joint probability distribution of $X_{(r)}$ and $W_{rs} = X_{(s)} - X_{(r)}$; $(r < s)$ based on a random sample of size n from the exponential distribution with parameter α.

(b) If X has a F-distribution with ν_1 and ν_2 degrees of freedom, find the distribution of $\frac{1}{X}$.

(c) Show that the m.g.f. of $Y = \log \chi^2$, where χ^2 follows Chi-square distribution with n d.f. is

$$M_Y(t) = \frac{2^t \Gamma\left(\frac{n}{2} + t\right)}{\Gamma(n/2)}$$

(d) Suppose a person is interested in testing the equality of two population standard deviations, say σ_1 and σ_2. For this purpose two samples of sizes n_1 and n_2 are drawn from the two populations respectively and suppose that the sample standard deviations are S_1 and S_2 respectively.
Explain how you would test the hypothesis $H_0 : \sigma_1 = \sigma_2$. Also discuss test of H_0 when both n_1 and n_2 are large.

(e) Show that for large degrees of freedom, t-distribution tends to standard normal distribution.

Answer the following questions : 10×3=30

4. (a) (i) Explain clearly the procedure generally followed in testing of a hypothesis. Also point the difference between one-tail and two-tail tests. 5

(ii) Show that in odd sample of size n from $U(0, 1)$ population, mean of the distribution of median is $\frac{1}{2}$. 5

Or

(b) Derive the probability density function of the student's t-distribution with ν d.f. and hence find its mean and variance. 10

5. (a) (i) Show that for large d.f., the Chi-square distribution tends to the normal distribution. 5