Total number of printed pages–6

3 (Sem–4/CBCS) PHY HC 1

2021

PHYSICS

(Honours)

Paper : PHY–HC–4016

(Mathematical Physics – III)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

GROUP–A

(Full Marks : 30)

1. Answer the following questions : 1×5= 5

   (a) Write the polar form of a complex number.

   (b) State Convolution theorem on Fourier transform.

Contd.

(c) Prove $L(1)=\frac{1}{s}$, $s>0$, using the definition of Laplace transform.

(d) Find the Residue of the complex function $f(z)=\frac{1}{1+z^2}$ at the pole $z=i$.

(e) Write the law of transformation for the tensor A_n^{lm}.

2. Answer the following questions : 2×5=10

(a) Find the modulus and argument of $-3i$.

(b) Check whether the function $f(Z)=ReZ$ is analytic or not.

(c) Write the Fourier's sine and cosine transform.

(d) If $L[f(x)]=\bar{f}(s)$, then show that $L\left[e^{ax}f(x)\right]=\bar{f}(s-a)$·

(e) Prove that a symmetric tensor of rank 2 in N dimensional space has $\frac{N(N+1)}{2}$ independent elements.

3. Answer ***any three*** of the following questions: 5×3=15

(a) Prove the Cauchy's integral formula —

$$f(a) = \frac{1}{2\pi i} \int_C \frac{f(z)}{z-a} \, dz$$

(b) Using Cauchy's residue theorem, show that —

$$\int_0^{\infty} \frac{dx}{1+x^2} = \frac{\pi}{2}$$

(c) Find the Fourier transform of the function —

$$f(x) = \begin{cases} 1, & \text{for } |x| < a \\ 0, & \text{for } |x| > a \end{cases}$$

(d) Find the Laplace transform of $f(t)$, where,

$$f(t) = \begin{cases} \sin t, & 0 < t < \pi \\ 0, & t > \pi \end{cases}$$

(e) What is Kronecker delta ? Show that it is a mixed tensor of rank 2.

GROUP–B

(Full Marks : 30)

4. Answer ***any three*** of the following questions :

(a) *(i)* Obtain the Cauchy-Reimann conditions for the function $f(z) = u + iv$ to be analytic, in a domain, u and v are functions of x and y. Are the conditions sufficient ? 6+1 =7

(ii) Expand the following function in Taylor series :

$$f(z) = \frac{1}{z+1}, \text{ about } z = 1.$$ 3

(b) Evaluate the following integrals using calculus of residues : 5×2=10

(i) $\int_{-\infty}^{+\infty} \frac{\sin x}{x} dx$

(ii) $\int_{0}^{2\pi} \frac{d\theta}{5 - 4\sin\theta}$

(c) Using Fourier transform, solve the one-dimensional heat flow equation $\frac{\partial u(x,t)}{\partial t} = \frac{\partial^2 u(x,t)}{\partial x^2}$, $x > 0$, $t > 0$, subject to the conditions –

(i) $u(0,t) = 0$

(ii) $u(x,0) = \begin{cases} 1, & 0 < x < 1 \\ 0, & x \geq 1 \end{cases}$

(iii) $u(x,t)$ is bounded. 10

(d) Evaluate the following inverse Laplace transforms – 4+6=10

(i) $L^{-1}\left[\frac{s^2}{(s+1)^2}\right]$

(ii) $L^{-1}\left[\frac{4P+5}{(P-4)^2(P+3)}\right]$

(e) (i) What is Levi-Civita tensor ? Prove that $\delta_{ij} \in_{ijk} = 0$. 2+1=3

(ii) Prove that

$\vec{A} \cdot \vec{B} = \sum_{i=1}^{3} \sum_{j=1}^{3} A_i B_j \delta_{ij}$. 3

(iii) Prove that tensor of rank 2 could be written as a sum of symmetric and asymmetric tensor. 4