Total number of printed pages–7

3 (Sem–4/CBCS) STA HC 1

2021

STATISTICS

(Honours)

Paper : STA–HC–4016

(Statistical Inference)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

GROUP–A

(Marks 30)

1. Answer the following questions as directed : $1 \times 5 = 5$

   (a) Maximum likelihood estimators (MLEs) are not necessarily unbiased.
   *(State True **or** False)*

Contd.

(b) The level of significance α of a test is also known as ———— of the critical region. *(Fill in the blank)*

(c) Power of the test is greater than and equal to —

(i) size of the critical region

(ii) (size of the critical region) -1

(iii) (size of the critical region) $+1$

(iv) None of the above

(Choose the correct option)

(d) In sampling from a normal population, the sample mean $\overline{x}$ is the most efficient estimator of the population mean μ.

*(State true **or** false)*

(e) State the Lehmann-Scheffe theorem.

2. Answer the following questions briefly : 2×5=10

(a) "Let $x_1, x_2, x_3, \ldots\ldots x_n$ be independent and identically distributed (IID) normal variables with common mean μ and common variance σ^2, then the hypothesis $H_0 : \mu = 10$ is a composite hypothesis". State whether the above statement is true or false, giving reasons in support of your answer.

(b) State the necessary and sufficient condition for a distribution to admit sufficient statistic.

(c) State the advantags and disadvantages of method of moments.

(d) Define Uniformly most powerful test.

(e) Estimate λ for the distribution

$$f(x,\lambda) = \frac{2}{\lambda^2}(\lambda - x), 0 < x < \lambda$$

for sample of size one.

3. Answer ***any three*** questions from the following : 5×3=15

(a) If T_1 and T_2 are two unbiased estimators of $\gamma(\theta)$, having the same variance and ρ is the correlation between them, then show that $\rho \geq 2e - 1$, where e is the efficiency of each estimator.

(b) State the Neyman-Pearson lemma and also explain the likelihood ratio test. When does the likelihood ratio principle lead to the same test as given by the Neyman-Pearson lemma ? 2+2+1=5

(c) Let p be the probability that a coin will fall in head in single toss in order to test $H_0 : p = 1/2$ against $H_1 : p = 1/4$. The coin is tossed 5 times and H_0 is rejected, if more than 3 heads are obtained. Find the probability of type-I error and the power of the test.

(d) Explain briefly the method of minimum Chi-square.

(e) Given the probability density function :

$$f(x,\theta)=\frac{1}{\pi\left\{1+(x-\theta)^2\right\}}, \quad -\infty < x < \infty, \quad -\infty < \theta < \theta$$

Find the Cramer-Rao lower bound of variance of an unbiased estimator of the parameter θ.

GROUP–B

(Marks 30)

4. Answer ***any three*** questions from the following : 10×3=30

(a) (i) Write a note on principle of maximum likelihood estimation. 3

(ii) In a city, the milk consumption of the families is assumed to be exponentially distributed with the parameter λ. The hypothesis $H_0 : \lambda = 5$ is rejected in favour of $H_1 : \lambda = 10$, if a family selected at random consumes 15 units or more. Obtain the critical region and the size of both the errors. 1+3+3=7

(b) (i) Let $x_1, x_2, x_3, \ldots\ldots x_n$ be a random sample drawn from a Poisson population with parameter λ. Test whether $T = \sum x_i$ is complete. 7

(ii) State the implication of Rao-Blackwell theorem. 3

(c) (i) Discuss briefly about Wald's Sequential Probability Ratio Test. State few important properties of the test. 3+3=6

(ii) With the help of an example, prove that an estimator which is unbiased, is also consistent for a parameter of a certain distribution. 4

(d) (i) Let $x_1, x_2, x_3, \ldots\ldots x_n$ be a random sample drawn from an exponential distribution with mean θ. Check the consistency of the estimator $T_n = n X_{(1)}$ for the parameter θ, where $X_{(1)}$ is the first ordered statistic. 7

(ii) "Unbiased estimators may not be unique". Justify with the help of an example. 3

(e) *(i)* Let $x_1, x_2, x_3, \ldots\ldots x_n$ be a random sample drawn from normal population with mean μ and variance σ^2. Explain the likelihood ratio test procedure to test $H_0 : \sigma^2 = \sigma_0^2$ (specified) against $H_1 : \sigma^2 \neq \sigma_0^2$ (specified). 7

(ii) Find the maximum likelihood estimator of θ for the following density — 3

$$f(x,\theta) = \theta^x (1-\theta)^{1-x} ; 0 \le \theta \le 1, x = 0 \text{ or } 1$$
