3 (Sem–1/CBCS) STA HC 2

2020

(Held in 2021)

STATISTICS

(Honours)

Paper : STA-HC-1026

(Calculus)

Full Marks : 80

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the questions as directed : 1×10=10

(a) The value of $\lim_{x \to 0} \frac{\sin x}{\sqrt{x}}$ is

(i) 0

(ii) 1

(iii) None of the above

(Choose the correct option)

(b) The general solution of the linear differential equation

$$\frac{d^2y}{dx^2} + p_1\frac{dy}{dx} + p_2 y = 0$$ has two

equal roots, then complementary function is

$$y = (c_1 + c_2 x)\, e^{ax}$$

*(State True **or** False)*

(c) What is the relation between gamma and beta function ?

(d) The value of $D^n e^{ax}$ is

(i) $a^n e^{ax}$

(ii) $\left(e^{ax}\right)^n$

(iii) $n\, e^{ax}$

(Choose the correct option)

(e) The stationary point which are not extreme points are called __________.

(Fill in the blank)

(f) State Euler's theorem on homogeneous function.

(g) If $f(x) = x^4 + x^2y^2 + y^4$, find f_x, f_{yx}.

(h) The differential equation

$$f(x,y)\left(\frac{d^m y}{dx^n}\right)^p + \phi(x,y)\left(\frac{d^{m-1} y}{dx^{m-1}}\right) + \ldots = 0$$

is of order _______ and degree _______.

(Fill in the blanks)

(i) Define integrating factor.

(j) Evaluate —

$$\lim_{x \to 0} \frac{a^x - b^x}{x}$$

2. Answer the following questions : 2×5=10

(a) Prove that

$$\beta(n,n) = \frac{\sqrt{\pi}\,\Gamma(n)}{2^{2n-1}\,\Gamma\left(n+\frac{1}{2}\right)}$$

(b) If $f(x,y) = \begin{cases} \dfrac{xy}{x^2+y^2} & , \ (x,y) \neq (0,0) \\ 0 & , \ (x,y) = (0,0) \end{cases}$

show that both the partial derivatives exist at (0,0) but the function is not continuous thereat.

(c) Find the maximum and minimum value of $f(x) = a\sin^2 x + b\cos^2 x$.

(d) Solve $x(y^2+1)dx + y(x^2+1)dy = 0$.

(e) Obtain the differential equation of the family of curves represented by

$y = e^x (A\cos x + B\sin x)$

where A and B are arbitrary constants.

3. Answer ***any four*** from the following questions : 5×4=20

(a) Evaluate

$$\lim_{x \to 0} (\cos x)^{\frac{1}{x^2}}$$

(b) By using the transformation $x+y=u,\ y=uv$, show that

$$\int_0^1 \int_0^{1-x} e^{\frac{y}{x+y}}\, dx\, dy = \frac{1}{2}(e-1)$$

(c) Reduce the equation

$(px-y)(x-py)=2p$ to Clairaut's form by the transformation $x^2=u, y^2=v$ and find its complete solution.

(d) If $u = log\, \frac{x^4+y^4}{x+y}$, show that

$$x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=3$$

(e) If $u_1=\frac{x_2x_3}{x_1}, u_2=\frac{x_3x_1}{x_2}, u_3=\frac{x_1x_2}{x_3}$

prove that $J(u_1, u_2, u_3)=4$.

(f) Solve the differential equation

$$(1+y^2)dx=(tan^{-1}y-x)dy$$

4. Answer ***any four*** of the following questions : 10×4=40

(a) *(i)* Consider the function

$$f(x) = (x-a)\sin\left(\frac{1}{x-a}\right); \; x \neq 0$$
$$= 0 \qquad ; \; x = a$$

Show that $f(x)$ is continuous but not derivable at $x = a$. 6

(ii) Prove that 4

$$\Gamma\left(\frac{3}{2}+x\right)\Gamma\left(\frac{3}{2}-x\right) = \left(\frac{1}{4}-x^2\right)\pi \sec \pi x,$$
$$-1 < 2x < 1$$

(b) *(i)* Find the integrating factor of differential equation and solve

$$\left(1+x^2\right)\frac{dy}{dx} + 2xy = \cos x \qquad 5$$

(ii) If $u = f(y-z, z-x, x-y)$, prove that 5

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$$

(c) (i) Evaluate $\iint (x+y)^2 \, dx \, dy$ over the area bounded by the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 .$$ 8

(ii) Prove that $\int_0^{\pi/2} \sqrt{\tan\theta} \; d\theta = \frac{\pi}{\sqrt{2}}$. 2

(d) The roots of the equation

$(\lambda - x)^3 + (\lambda - y)^3 + (\lambda - z)^3 = 0$ in λ are u, v, w. Prove that

$$\frac{\partial(u, v, w)}{\partial(x, y, z)} = -2\frac{(y-z)(z-x)(x-y)}{(v-w)(w-u)(u-v)}$$

10

(e) (i) Express $\int_0^{\pi/2} \sin^4\theta \cos^6\theta \, d\theta$ as a beta function and hence evaluate it. 2+3=5

(ii) Solve

$y = 3x + \log p$ 5

(f) (i) Dicuss the derivability of the following function

$f(x) = 2x - 3\ ;\ 0 \le x \le 2$

$= x^2 - 3\ ;\ 2 < x \le 4$

at the point $x=4$. 5

(ii) Solve $\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = sin 2x$ 5

(g) (i) Solve the partial differential equation $x^2p + y^2p = z^2$ 5

(ii) Show that $\int_0^1 \frac{dx}{(1-x^6)^{\frac{1}{6}}} = \frac{\pi}{3}$ 5

(h) Show that the volume of the greatest rectangular parallelepiped that can be inscribed in the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ is $8abc / 3\sqrt{3}$. 10