Or

With a suitable diagram illustrate the Stern-Gerlach experiment. What is the significance of inhomogeneous magnetic field used in Stern-Gerlach experiment ? explain mathematically.

In a Stern-Gerlach type experiment, the magnetic field varies with distance in z-direction according to $dB_z/dz = 1{\cdot}4\ T/mm$. Silver atoms travel a distance $x = 3{\cdot}5\ cm$ through the magnet. The speed of atoms emerging from oven is $v = 750\ m/sec$. Find the separation of the two beams as they leave the magnet. Mass of silver atom $= 1{\cdot}8 \times 10^{-25}\ kg$ and its magnetic moment is 1 *Bohr* magneton.

3+3+4=10

3 (Sem–5 /CBCS) PHY HC 1

2021

(Held in 2022)

PHYSICS

(Honours)

Paper : PHY–HC–5016

(Quantum Mechanics and Applications)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions : 1×7=7

(a) Write down the quantum mechanical form of total energy operator of a particle moving in *x*-direction.

(b) Determine whether or not $\psi(x) = e^x$ is an acceptable wave function.

(c) Show that $\left[x, \frac{\delta^2}{\delta x^2} \right] = -2 \frac{\delta}{\delta x}$

(d) How does the number of superimposed waves, forming a wave packet, affect the localization of the particle ?

(e) What is Landé g-factor ?

(f) What is the total number of energy level (or degeneracy) for the nth state of hydrogen atom ?

(g) The splitting of a spectral line in the presence of external electric field is termed as

(i) Anomalous Zeeman effect

(ii) Paschen-Back effect

(iii) Normal Zeeman effect

(iv) Stark effect

2. Answer the following questions : 2×4=8

(a) What is the physical significance of the wave function $\psi(x, t)$?

(b) A particle with total energy E is influenced by a potential energy $V(x)$. Show that the one-dimensional Schrödinger equation can be written in the form

$$\left[\frac{d^2}{dx^2} + k^2 - U(x) \right] \psi(x) = 0$$

where,

$$k^2 = \frac{2mE}{\hbar^2} \quad \text{and} \quad U(x) = \frac{2mV(x)}{\hbar^2}$$

(c) Show that $\hat{p}_x = -i\hbar \frac{d}{d\phi}$ is an Hermitian operator.

(d) A particle of mass m and moving in a potential $v(x)$ has the wave function

$$\psi(x,t) = A \exp\left(-ikt - \frac{km}{\hbar} x^2 \right)$$

where both A and k are constants. Determine the explicit form of the potential.

Contd.

3. Answer ***any three*** of the following : 5×3=15

(a) What do you mean by expectation value of a dynamical variable ? Find the expectation value $<p>$ and $<p^2>$ for the wave function

$$\psi(x) = \sqrt{2/L}\, sin\left(\frac{\pi x}{L}\right) \quad \text{for } 0<|x|<L$$
$$\quad \text{for } |x|>L$$

1+(2+2)=5

(b) Show that if $\psi_1(\bar{r})$ and $\psi_2(\bar{r})$ are two independent solutions of the Schrödinger equation, then

$$\psi(\bar{r}) = a_1\psi_1(\bar{r}) + a_2\psi_2(\bar{r})$$

is also a solution of the Schrödinger equation. What does it imply ? 4+1=5

(c) What are momentum space wave functions ? Show that these wave functions can be obtained as Fourier transform of position space wave functions. 1+4=5

(d) Write down the radial wave function for 1s state of hydrogen atom. Also, compare the probabilities of a 1s electron in the hydrogen atom being at a distance a_0 from the nucleus than at a distance $a_0/2$. 1+4=5

(e) State Pauli's exclusion principle. An atomic state is denoted by $^4D_{5/2}$. Give the values of *L*, *S* and *J*. What should be the minimum number of electrons involved for the state ? 2+3=5

4. What is the need for normalization of a wave function ? Calculate the normalization constant of a wave function (at $t=0$) given by

$$\psi(x) = a\,e^{\left(-a^2x^2/2\right)}\, e^{ikx}$$

known as the Gaussian wave packet.

Determine *(a)* the probability density, and *(b)* the probability current density of the wave function. 2+3+5=10

OR

A finite square potential well of depth V_0 is defined as

$$V(x) = \begin{cases} 0 & \text{for } x < 0 \\ -V_0 & \text{for } 0 \le x \le L \\ 0 & \text{for } x > L \end{cases}$$

Set up Schrödinger equation for the potential well. Also solve using appropriate boundary conditions and determine the energy eigenvalues. 2+8=10

5. (i) Write down Schrödinger equation for a linear harmonic oscillator. What are the eigen-values and eigen-functions of the Hamiltonian of a linear harmonic oscillator ? Explain the significance of zero-point energy of the oscillator. 1+2+2=5

(ii) Find the expectation value of energy when the state of harmonic oscillator is described by the following wave function :

$$\psi(x,t) = \frac{1}{\sqrt{2}}\left[\psi_0(x,t) + \psi_1(x,t)\right]$$

where $\psi_0(x,t)$ and $\psi_1(x,t)$ are wave functions for the ground state and the first excited state respectively. 5

Or

Write down Schrödinger wave equation for hydrogen atom in spherical polar coordinates. Separate the equation into radial and two angular parts. Also, from the radial part of the Schrödinger equation, find the eigenvalues of energy E for the ground state of hydrogen atom. 1+2+7=10

6. (i) Describe and explain L-S coupling. Under what condition does it hold ?

(ii) Under what condition L-S coupling breaks down and what kind of new coupling takes place ?

(iii) Describe J-J coupling. Illustrate L-S and J-J coupling with the help of vector diagram. 3+3+4=10