Total number of printed pages–7

3 (Sem–5/CBCS) PHY HE3

2021

(Held in 2022)

PHYSICS

(Honours Elective)

Paper : PHY–HE–5036

(Advanced Mathematical Physics)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions : 1×7=7

   (a) Define the linear vector space.

   (b) What is field ? Give *two* examples.

   (c) Show that $(A+B)(A-B)=A^2-B^2$ if and only if A and B commute.

Contd.

(d) Define quotient law of tensors.

(e) Give the transformation equation for the tensor A_{st}^{lmn} and write its rank.

(f) What is Minkowski space?

(g) What do you mean by basis in a linear vector space?

2. Answer the following questions : 2×4=8

(a) if $\vec{x}, \vec{y}$ and $\vec{z}$ are linearly independent vectors, determine whether $\vec{x}+\vec{y}, \vec{y}+\vec{z}$ and $\vec{z}+\vec{x}$ are linearly dependent or independent.

(b) In a vector space $V(F)$, for all $v \in V$ and $f \in F$, prove that

(i) $\theta = 0.v$

(ii) $(-f)v = f(-v) = -(fv)$

(c) Diagonalize the matrix $\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$.

(d) Show that $div\left(curl\ \vec{F}\right) = 0$ for a vector $\vec{F}$ using tensor analysis.

3. Answer **any three** of the following questions : 5×3=15

(a) Find whether the set of vectors $[x_1, y_1, z_1]$ in R^3, such that $x_1 + y_1 + z_1 = 0$ forms a subspace of R^3.

(b) Show that every linearly independent vector belonging to a vector space has a unique representation as a linear combination of its base vector.

(c) Find the vector associated with the given second-order antisymmetric tensor

$$\begin{bmatrix} 0 & (x+y+z) & -(x+y) \\ -(x+y+z) & 0 & x \\ (x+y) & -x & 0 \end{bmatrix}$$

(d) Define Kronecker delta. Show that it is a mixed tensor of rank 2.

(e) What is alternating tensor? Prove that

$$\varepsilon_{iks}\varepsilon_{mps} = \delta_{im}\delta_{kp} - \delta_{ip}\delta_{km} = 0.$$

4. Answer ***any three*** of the following questions : 10×3=30

(a) (i) Prove that the set I of all the integers with the binary operation $*$ defined by $a * b = a + b - 1$ form a group. 4

(ii) Determine e^A, when $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$. 6

(b) (i) Verify Cayley-Hamilton theorem for the matrix $\begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$. Hence find A^{-1}. 3+1=4

(ii) Solve the coupled homogeneous linear differential equation using matrix method

$$\frac{dy_1}{dt} = 3y_1 + y_2$$

$$\frac{dy_2}{dt} = y_1 + 3y_2$$

subject to initial conditions

$y_1(0) = 3;\ y_2(0) = 5$. 6

(c) Using tensors, prove the following identities : 3+3+4=10

(i) $\vec{\nabla} \times (\phi \vec{A}) = \phi(\vec{\nabla} \times \vec{A}) + (\vec{\nabla}\phi) \times \vec{A}$

(ii) $\vec{\nabla}\times\left(\vec{\nabla}\times\vec{A}\right)=\vec{\nabla}\left(\vec{\nabla}\cdot\vec{A}\right)-\nabla^2\vec{A}$

(iii) $\vec{\nabla}\times\left(\vec{A}\times\vec{B}\right)=\vec{A}\left(\vec{\nabla}\cdot\vec{B}\right)-\vec{B}\left(\vec{\nabla}\cdot\vec{A}\right)$

$$-\left(\vec{A}\cdot\vec{\nabla}\right)\vec{B}+\left(\vec{B}\cdot\vec{\nabla}\right)\vec{A}$$

(d) (i) Write the tensor form of the angle between two lines with direction cosines l_i, m_i and also write the condition for the two lines to be co-planar. 3+3=6

(ii) Define moment of inertia tensor. Show that it is a symmetric tensor of order 2. 1+3=4

(e) (i) Find the eigenvalue and eigenvectors of the matrix

$$A=\begin{bmatrix}0 & 1 & 0\\ 1 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$$ 6

(ii) If H matrix is Hermitian, and I is identity matrix, show that $(H-iI)(H+iI)^{-1}$ is a unitary matrix. 4

3500