2019

PHYSICS

(Major)

Paper : 1.1

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

GROUP—A

(Mathematical Method)

(*Marks* : 20)

1. *(a)* Define scalar field and vector field in a region R in space. 1

(b) If $\phi(r)$ is a scalar field, state whether the two expressions below are scalar or vector : 1

(i) $\nabla^2\phi(r)$

(ii) $\nabla^2[\vec{\nabla}\phi(r)]$

2. *(a)* Prove that if the scalar triple product of three vectors vanishes, then the vectors are coplanar. 2

(b) If $\vec{A}$ and $\vec{B}$ are irrotational, then prove that $\vec{A} \times \vec{B}$ is solenoidal. 2

(c) What is the physical significance of grade $\vec{A}$? 2

(d) Some scalar field is given by

$$\phi(r) = r^2 = x^2 + y^2 + z^2$$

Show that $\vec{\nabla} r$ is a unit vector. 2

3. Answer any *two* questions : 5×2=10

(a) Show that the unit vector perpendicular to each of the vectors $\vec{A} = 3\hat{i} + \hat{j} + 2\hat{k}$ and $\vec{B} = 2\hat{i} - 2\hat{j} + 4\hat{k}$ is $\frac{1}{\sqrt{3}}(\hat{i} - \hat{j} - \hat{k})$ and that the sine of the angle between them is $\frac{2}{\sqrt{7}}$.

(b) Find $\vec{\nabla} \cdot \vec{F}$ and $\vec{\nabla} \times \vec{F}$, where

$$\vec{F} = \vec{\nabla}(x^3 + y^3 + z^3 - 3xyz)$$

(c) Show that the curl of the linear velocity $\vec{v}$ of a particle of a rigid body is equal to twice the angular velocity $\vec{\omega}$.

GROUP—B

(Mechanics)

(*Marks* : 40)

4. *(a)* What is the difference between laboratory frame of reference and centre of mass frame of reference? 1

(b) What is an equipotential surface? Can we have equipotential surface for gravitational field of unit mass? 1

(c) What is the rotational analogue of the mass of a body? Is it a vector quantity? 1

(d) Newton's laws of motion are not valid in non-inertial frame of reference. Explain why. 1

(e) Name the fictitious forces obtained in the rotating frame of reference. 1

(f) Name the force which is required to keep the satellite in the orbits. Is it a conservative force? 1

5. *(a)* A particle of mass m_1 moving with a velocity u_1 is elastically scattered from another particle of mass m_2 at rest. After collision two particles move in opposite directions with same speed. Find the relation between the two masses. 2

(b) Distinguish between inertial mass and gravitational mass. Are gravitational mass and inertial mass of a body identical? 2

6. Answer any *two* questions : 5×2=10

(a) Show that when the vector sum of external forces acting upon a system equals zero, the total linear momentum of the system is conserved.

(b) Show that the force

$$\vec{F} = (y^2 - 2xyz^3)\hat{i} + (3 + 2xy - x^2z^3)\hat{j} + (6z^3 - 3x^2yz^2)\hat{k}$$

is a conservative force.

(c) Explain how the acceleration due to gravity is determined by Kater's pendulum in the laboratory.

7. Answer any *two* questions : 10×2=20

(a) Establish work energy theorem. Show that for conservative force field, the sum of potential energy and kinetic energy of a particle remains the same. 5+5=10

(b) Deduce expressions for the gravitational potential and attraction due to a thin uniform spherical shell at a general point outside as well as inside the shell. Give a graphical representation also. 4+4+2=10

(c) What are moment of inertia and radius of gyration of a rotating body? Is moment of inertia a vector or a scalar quantity? What is the physical significance of moment of inertia? State and prove the theorem of perpendicular axis of moment of inertia. 2+1+2+5=10

(d) Show mathematically that the Coriolis force and centrifugal force are produced as a result of motion of the particles in the rotating coordinate system.

Discuss how the gravitational acceleration is affected when a particle of mass is placed at any point on the earth surface. 7+3=10

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