Total number of printed pages–7

**3 (Sem–1 /CBCS) PHY HC 1**

**2021**

**( Held in 2022 )**

**PHYSICS**

(Honours)

Paper : PHY–HC–1016

***( Mathematical Physics–I )***

*Full Marks : 60*

Time : Three hours

***The figures in the margin indicate full marks for the questions.***

1. Answer the following questions : 1×7=7

   *(a)* State the vector field with respect to Cartesian co-ordinate. Give *one* example.

   *(b)* Show that $\vec{\nabla}.\vec{r}=3$, where $\vec{r}=\hat{i}x+\hat{j}y+\hat{k}z$.

*Contd.*

(c) Write the order and degree of the differential equation

$$x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+y=0$$

(d) Write the volume element in curvilinear co-ordinate.

(e) Give the value of $\int_{-\alpha}^{+\alpha}\delta(x)\,dx$

(f) Define variance in statistics.

(g) State the principle of least square fit.

2. Answer of the following questions : 2×4=8

(a) Find a unit vector perpendicular to the surface, $x^2+y^2-z^2=11$ at the point $(4,2,3)$.

(b) If $\vec{A}=\vec{A}(t)$, then show that

$$\frac{d}{dt}\left[\vec{A}.\left(\frac{d\vec{A}}{dt}\times\frac{d^2\vec{A}}{dt^2}\right)\right]=A.\left[\frac{d\vec{A}}{dt}\times\frac{d^3\vec{A}}{dt^3}\right]$$



(c) If $\vec{A}$ and $\vec{B}$ are each irrotational, prove that $\vec{A}\times\vec{B}$ is solenoidal.

(d) Evaluate $\iint_S \bar{r}\times\hat{n}dS$, where $S$ is a closed surface.

3. Answer **any three** of the following questions : 5×3=15

(a) Prove

$$\iiint_V\left(\phi\nabla^2\psi-\psi\nabla^2\phi\right)dV=\iint_S\left(\phi\vec{\nabla}\psi-\psi\vec{\nabla}\phi\right).dS$$

(b) Find the integrating factor (IF) of the following differential equation and solve it.

$$\left(1+x^2\right)\frac{dy}{dx}+2xy=\cos x$$

(c) Express curl $\vec{A} = \vec{\nabla} \times \vec{A}$ in cylindrical co-ordinate.

(d) What is Dirac-delta function ? Show that the function

$$\delta(x) = \lim_{\varepsilon \to 0} \frac{\sin(2\pi\varepsilon x)}{\pi\varepsilon}$$

is a Dirac delta function.

(e) If $\phi(x,y,z) = 3x^2y - y^3x^2$ be any scalar function $\phi$, find out

(i) grad $\phi$ at point (1, 2, 2)

(ii) unit vector $\hat{e}$ perpendicular to surface.

4. Answer ***any three*** of the following questions : 10×3=30

(a) (i) If $F_1(x,y)$, $F_2(x,y)$ are two continuous functions having continuous partial derivatives $\frac{\partial F_1}{\partial y}$ and $\frac{\partial F_2}{\partial x}$ over a region $R$ bounded by simple closed curve $C$ in the $x$–$y$ plane, then show that

$$\oint_C (F_1dx + F_2dy) = \iint_S \left(\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y}\right) dx\,dy$$ 7



(ii) A function $f(x)$ is defined

$$\text{as} \begin{cases} 0, \; x<2 \\ \frac{1}{18}(2x+3), \; 2 \le x \le 4 \\ 0, \; x>2 \end{cases}$$

Show that it is a probability density function. 3

(b) Solve the following differential equations : 5+5=10

(i) $9\frac{d^2y}{dx^2} + 12\frac{dy}{dx} + 4y = 6e^{-2x/3}$

(ii) $2\frac{\partial^2 z}{\partial x^2} + 5\frac{\partial^2 z}{\partial x \partial y} + 2\frac{\partial^2 z}{\partial y^2} = 0$

(c) (i) A rigid body rotates about an axis passing through the origin with angular velocity $\vec{\omega}$ and with linear velocity $\vec{v} = \vec{\omega} \times \vec{r}$, then prove that,

$$\vec{\omega} = \frac{1}{2}\left(\vec{\nabla} \times \vec{v}\right)$$

where, $\vec{\omega} = \hat{i}\omega_1 + \hat{j}\omega_2 + \hat{k}\omega_3$

$\vec{r} = \hat{i}x + \hat{j}y + \hat{k}z$ 5

(ii) If $y = f(x+at) + g(x-at)$, show that it satisfies the equation

$$\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}$$

where $f$ and $g$ are assumed to be at least twice differentiable and $a$ is any constant. 5

(d) (i) Apply Green's theorem in plane to evaluate the integral

$$\oint_C \left[ (xy - x^2)\, dx + x^2 y\, dy \right]$$ over the triangle bounded by the line $y=0$, $x=1$ and $y=x$. 6

(ii) Prove that

$$\int_{-\alpha}^{+\alpha} f(x)\, \delta(x-c)\, dx = f(c)$$ 4



(e) (i) Applying Gauss' theorem, evaluate

$$\iint_S x\, dydz + y\, dzdx + z\, dxdy,$$ where

$S$ is the sphere of radius $x^2 + y^2 + z^2 = 1$ 5

(ii) Evaluate $\nabla^2 \psi$ in spherical co-ordinate. 5

---