3 (Sem–6/CBCS) MAT HC 2

[redacted]

2025

MATHEMATICS

(Honours Core)

Paper : MAT–HC–6026

(Partial Differential Equations)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following as directed : 1×7=7

   (i) Which of the following methods can be used to construct a first-order partial differential equation?

      (a) By differentiating a given function with respect to multiple independent variables

      (b) By eliminating one or more arbitrary constants from a given relation

(c) By integrating a given function with respect to the dependent variable

(d) None of the above

(Choose the correct answer)

(ii) Along every characteristic strip of the equation $F(x, y, z, p, q) = 0$, the function $F(x, y, z, p, q)$ is ______.

(Fill in the blank)

(iii) Charpit's method can be applied to both linear and nonlinear first-order partial differential equations.

(State True **or** False)

(iv) What is the primary goal of transforming a first-order linear PDE into its canonical form?

(a) To simplify the equation and make it easier to solve, often using characteristic curves

(b) To eliminate the need for the method of characteristics

(c) To ensure the equation has only one variable

(d) To convert the equation into a second-order PDE.

(Choose the correct answer)

(v) In the method of separation of variables, we assume a solution of the form $u(x, y) = X(x)Y(y)$, leading to two ODEs. The constant λ that arises from separation is known as the ______ constant. (Fill in the blank)

(vi) Which of the following is a characteristic of a hyperbolic second-order linear partial differential equation?

(a) It describes steady-state phenomena

(b) It describes systems in equilibrium

(c) It models wave propagation

(d) It has a solution that does not change over time

(Choose the correct answer)

(vii) The general solution of a linear second-order partial differential equation with constant coefficients is the sum of the ______ (the solution to the corresponding homogeneous equation) and the particular integral (a solution to the non-homogeneous equation).

(Fill in the blank)

2. Answer in short : 2×4=8

(i) Define first-order quasi-linear and semi-linear partial differential equations.

(ii) Construct the first-order partial differential equation for the family of surfaces defined by $z = x^2 + y^2 + xy + C$, where C is a constant.

(iii) State the basic idea behind Cauchy's method of characteristics for solving nonlinear first-order partial differential equations.

(iv) Determine whether the following equation is parabolic, elliptic or hyperbolic.

$$u_{xx} + x^2 u_{yy} = 0$$

3. Answer **any three** : 5×3=15

(i) Find the integral surface of the equation $x\left(y^2+z\right)p - y\left(x^2+z\right)q = \left(x^2-y^2\right)z$ which contains the straight line $x + y = 0, z = 1$.

(ii) Define the concept of 'general integral' of a first-order nonlinear partial differential equation. Explain it for the equation $z^2\left(1+p^2+q^2\right)=1$.

(iii) Reduce to canonical form and find the general solution of $u_x + xu_y = y$.

(iv) Apply $\sqrt{u} = v$ and $v(x, y) = f(x) + g(y)$ to solve the equation $x^4 u_x^2 + y^2 u_y^2 = 4u$.

(v) Find the characteristic curves and then reduce the equation
$$u_{xx} + (2\operatorname{cosec} y)u_{xy} + \left(\operatorname{cosec}^2 y\right)u_{yy} = 0$$
to the canonical form.

4. Answer the following : 10×3=30

(i) Find a complete integral of the equation $\left(p^2+q^2\right)x = pz$ and deduce the solution which passes through the curve $x = 0, z^2 = 4y$.

Or

Solve –

$$(p_1 + x_1)^2 + (p_2 + x_2)^2 + (p_3 + x_3)^2 = 3(x_1 + x_2 + x_3)$$

by Jacobi's method.

(ii) Apply the method of separation of variables $u(x, y) = f(x)g(y)$ to solve the equation $y^2u_x^2 + x^2u_y^2 = (xyu)^2$, $u(x, 0) = 3exp\left(\frac{x^2}{4}\right)$.

Or

Apply $v = ln\,u$ and then $v(x, y) = f(x) + g(y)$ to solve the equation $x^2u_x^2 + y^2u_y^2 = (xyu)^2$.

(iii) Determine the region in which the given equation is hyperbolic, parabolic, or elliptic, and transform the equation in the respective region to canonical form.

(a) $u_{xx} + xyu_{yy} = 0$

(b) $u_{xx} + u_{xy} - xu_{yy} = 0$

Or

Find the general solutions of the following equations :

(a) $x^2u_{xx} + 2xyu_{xy} + y^2u_{yy} = 0$

(b) $3u_{xx} + 10u_{xy} + 3u_{yy} = 0$

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