3(Sem-8/FYUGP)BNC(A)/DSCI

2025

MATHEMATICS

(Discipline Specific Core)

Paper Name: Abstract Algebra

Paper Code: **MAT-DSC-244**

Full Marks: 60

Time: Two and Half Hours

(*The figures in the margin indicate full marks for the questions)*

1. Answer the following questions: 1x7=7

a. Give an example of a non-abelian group whose every proper subgroup is abelian.

b. In a cyclic group of order 24, find the number of elements of order 6.

c. Give an example of an infinite non commutative ring that does not have unity.

d. Define maximal ideal of a ring.

e. What is the generator of the cyclic group of n^{th} roots of unity.

f. What is the characteristics of $\mathbb{Z}_p$, where p is prime.

g. Define normal subgroup.

2. Answer any four of the following questions: 2x4=8

a. Give an example to show that Union of two groups may not be group.

b. Show that the kernel of a group homomorphism is a normal subgroup.

c. List the units of $(\mathbb{Z}_9, +_9, \times_9)$.

d. Find the order of the group $U(10)$ and order of each element of $U(10)$.

e. State first Isomorphism theorem of groups.

3. Answer any three of the following questions: 5x3=15

a. Show that the Center of a group is a Normal Subgroup. Also, Write the Center of a Dihedral group $D_n, n \geq 3$.

b. Define Automorphism. Show that for a group G and a fixed element $g \in G,$ the function $\emptyset : G \to G,$ defined by $\emptyset_g(x) = gxg^{-1}, \forall\, x \in G$ is an automorphism.

c. Show that the ideal $< x^2 + 1 >$ of $\mathbb{R}[x]$ is a maximal ideal.

d. Prove that every permutation of a finite set can be written as a cycle or as a product of disjoint cycles.

e. Define Coset of a group. State and prove Lagrange's Theorem.

4. Answer any three of the following questions: 10x3=30

a. Let G be a group and $Z(G)$ be the center of G. If $\frac{G}{Z(G)}$ is cyclic, then prove that G is abelian. Also, prove that any group of order p^2 is abelian, where p is prime. $5 + 5 = 10$

b. Let R be a commutative ring with Unity, and I be an ideal of R. Then show that $\frac{R}{I}$ is a field if and only if I is a maximal ideal.

$5 + 5 = 10$

c. Show that the order of a cyclic group is equal to the order of its generator. Prove that converse of Lagrange's theorem holds in finite cyclic group. $6 + 4 = 10$

d. State and prove Cayley's theorem. 10

e. Show that if a group has order 10 then it must have a subgroup of order 5. Also, show that any subgroup of index 2 is a normal subgroup. 10

****