1 (Sem–4) PHY 4

2025

PHYSICS

Paper : PHY0400404

(Mathematical Physics)

Full Marks : 45

Time : Two hours

The figures in the margin indicate full marks for the questions.

1. Answer to the following questions : $1\times5=5$

   (a) Express the Laplace equation for two-dimensional Cartesian system.

   (b) What is the necessary condition for the existence of the Fourier's series ?

(c) If certain complex function $f(z)$ is analytic then which equation should it satisfy ?

(d) What is the residue of a function at a simple pole ?

(e) What is the fundamental property of Levi-Civita symbol, ε_{ijk} ?

2. Answer ***any five*** questions : 2×5=10

(a) Express the general form of Laplace equation in spherical polar coordinate system.

(b) State the Dirichlet conditions for the existence of a Fourier series.

(c) State the Cauchy-Riemann conditions in Cartesian coordinates.

(d) Express the difference between a pole and a branch point with examples.

(e) Define a symmetric tensor with an example.

(f) What is the significance of Einstein's summation convention ?

(g) Write down the probability mass function of the Poisson distribution and illustrate the parameters within.

(h) Find the mean and variance of a binomial distribution with parameters n and p.

(i) Express the Fourier series of a function $f(x) = x$ defined within the bound $(-\pi, \pi)$.

(j) What type of boundary conditions are used to solve the wave equation for a vibrating string fixed at both ends ?

3. Answer ***any four*** questions : 5×4=20

(a) Solve the one-dimensional wave equation for a string of length L fixed at both ends using the separation of variables method (Give the general form of the solution and mention the boundary conditions).

(b) Solve the Laplace's equation in two dimensions for a rectangular region with suitable boundary conditions using separation of variables method (express the solution graphically without evaluating the arbitrary constants).

(c) Find the Fourier series (sine and cosine form) for the function $f(x) = x^2$ in the interval $(-\pi, \pi)$. Show all steps clearly.

(d) Expand the periodic square wave function in a complex Fourier series and write down the expression.

(e) State and prove Cauchy's integral formula for a function analytic in a simply connected domain. (State all assumptions clearly)

(f) Determine the nature and the order of the singularity of the function,

$f(z) = \dfrac{\sin z}{z^3}$ at $z = 0$, and compute its residue.

(g) If $A^{\mu\nu}$ and $B_{\mu\nu}$ are two tensors of rank two, then describe their transformation rules under coordinate transformation and also show that $A^{\mu\nu}B_{\mu\nu}$ is an invariant quantity.

(h) Starting from the Poisson distribution, derive the condition under which it approximates the binomial distribution. Explain the physical or statistical significance of this limit.

4. Answer ***any one*** question : 10×1=10

(a) Solve the following differential equation using separation of variable method.

$$\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u, \text{ where } u(x, 0) = 6e^{-3x}.$$

(b) Given, $f(x) = \begin{cases} -1, & \text{for } -\pi < x < -\frac{\pi}{2}, \\ 0, & \text{for } -\frac{\pi}{2} < x < \frac{\pi}{2}, \\ +1, & \text{for } \frac{\pi}{2} < x < \pi \end{cases}$

find the Fourier expression of $f(x)$.

(c) Evaluate the integral $\oint_C \frac{e^z}{z^2(z-1)} dz$, where C is the positively oriented circle $|z| = 2$, using residue theorem. Clearly identify the singularities of the integrand, determine the order of each singularity, compute the residues at the poles enclosed by C. 6+4=10

(d) Prove the quotient law of tensors. Show that the Kronecker delta δ^i_j behaves as a mixed tensor. What is its rank? Using tensor transformation laws, show that it acts as the identity operator under index contraction. 4+3+1+2=10
