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1 (Sem-4) PHY 2

2025

PHYSICS

Paper : PHY0400204

(Quantum Mechanics)

Full Marks : 45

Time : 2 hours

The figures in the margin indicate full marks for the questions.

Symbols have their usual meanings.

1. Objective-type : (Answer **all** questions)

1×5=5

(a) If λ_c is the Compton shift, what is the greatest wavelength change in Compton scattering ?

(b) What is the outcome of $[\hat{x}, e^{\hat{x}}]$?

(c) Plot the wavefunction $\psi(x) = \frac{1}{a^2} x e^{-x/a}$ for $x > 0$, where a is a constant and real number.

(d) What is the requirement of de Broglie wavelength of electron for the diffraction of electrons by a crystal?

(e) What is the total degeneracy in energy of H -atom with principal quantum number $n = 2$?

2. Very short answer-type : (Answer **any five** questions) 2×5=10

(a) Write Planck's blackbody radiation formula and obtain Rayleigh-Jeans formula under limiting condition.

(b) Write the general form of the eigenvalue equations for the Hamiltonian of a one-dimensional linear harmonic oscillator and mention possible eigenvalues.

(c) What is the minimum value of the product $\Delta x \Delta p_x$? Plot Δp_x versus Δx . 1+1=2

(d) Why group velocity and not the phase velocity is considered to describe the velocity of a moving material particle?

(e) What is zero-point energy? Why it cannot be equal to zero for a particle confined in a potential box? 1+1=2

(f) If λ_p and λ_α are the de Broglie wavelengths of a proton and an alpha particle moving with same non-relativistic speeds, then find the ratio

$$\frac{\lambda_p}{\lambda_\alpha}$$

(g) Write the differential forms of linear momentum and energy operators. Is momentum operator Hermitian? 1+1=2

(h) An attempt is made to measure the position of an electron in an atom. The uncertainty of this measurement is $1\text{\AA}$. What is the minimum uncertainty in the measurement of linear momentum of the electron?

(i) Using the general expression for spherical harmonics $Y_l^{ml}(\theta, \varphi)$, evaluate Y_1^0 .

(j) Write the eigenvalue equations for the z-component of orbital angular momentum and square of the orbital angular momentum operators for a particle under the action of a spherically symmetric potential.

3. Short answer-type : (Answer **any four** questions) 5×4=20

(a) Obtain the normalization constant by normalizing the given wavefunction

$$\psi(x) = \frac{3}{\sqrt{10}}(a^2 - x^2) \text{ in the region}$$

$-a \leq x \leq a$. Hence, show the variation of the normalized wavefunction with x graphically with a mention of the peak value. 3+2=5

(b) An experiment on photoelectric effect is conducted for a metal. The stopping potentials are 4.50V and 0.20V corresponding to light wavelengths 190nm and 550nm, respectively. Find the work function of the metal.

(c) Provide a brief physical interpretation of wavefunction.

(d) Consider a beam of particles of mass m , moving in the positive x direction with energy E towards a potential step at $x = 0$. The potential $V(x)$ is zero for $x \leq 0$ and it is $\frac{3}{4}E$ for $x > 0$. Find the reflection coefficient.

(e) Starting with time dependent Schrödinger equation, obtain the differential form of equation of continuity involving the probability current density.

(f) Briefly describe the Davisson-Germer experiment that confirms wave nature of electrons.

(g) Write the time-independent Schrödinger equation for a one-dimensional linear harmonic oscillator and provide its ground state solution using the Hermite polynomials.

(h) Find the outcome of the commutation relation,

$$[\hat{x}\hat{p}_y - \hat{y}\hat{p}_x, \hat{y}\hat{p}_z - \hat{z}\hat{p}_y]$$

4. Essay-type : (Answer **any one** question)

10×1=10

(a) Starting with the concept of wave packet, obtain the intensity distribution. Introduce the Gaussian form of wave packet and briefly explain its connection with a moving material particle.

7+3=10

(b) Write the time independent Schrödinger equation in three dimensions for a particle experiencing central potential and obtain its radial and angular parts in spherical polar coordinate system. Using the idea of separation of variables, find the normalized azimuthal wavefunction.

7+3=10

(c) Consider a particle inside a one-dimensional potential box having infinite potential barriers at $x=0$ and $x=L$. The wavefunction of the particle is $\psi(x) = Nx(L-x)$, where N is the normalization constant. Find the expectation values of position and linear momentum operators.