(d) Mass of a relativistic particle changes with velocity as

$$m = \frac{m_0}{\sqrt{1 - v^2/C^2}}, \text{ where } m_0 \text{ is the}$$

rest mass. If velocity of the particle increases from 0 to v use work energy theorem to show that gain in kinetic energy of the particle is $E_k = (m - m_0)C^2$. From this show that total relativistic energy of the particle

is $E = \dfrac{m_0 C^2}{\sqrt{1 - v^2/C^2}}$. 8+2=10

(e) Show that Lorentz transformation reduces to Galilean transformation if $v << C$. Represent Lorentz transformation as rotation in spacetime. From Lorentz transformation equations for (x, y, z, t), show that

$$c^2 t'^2 - x'^2 - y'^2 - z'^2 = c^2 t^2 - x^2 - y^2 - z^2.$$

2+5+3=10

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PHYSICS

Paper : PHY0400104

(Classical Mechanics)

Full Marks : 60

Time : 2½ hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions : 1×8=8

(a) How many degrees of freedom are possessed by a ball moving on the surface of a sphere ?

(b) Lagrangian of a free particle moving along X-axis is given by $L = \frac{1}{2} m\dot{x}^2$. What is its generalised momentum ?

(c) Which one of the following is a correct expression for Legendre transformation ?

(i) $H = \sum \dot{p}_j q_j - L$

(ii) $H = \sum p_j \dot{q}_j + L$

(iii) $H = \sum p_j \dot{q}_j - L$

(iv) $H = \sum \dot{p}_j \dot{q}_j - L$

(d) Lagrangian of a particle moving in a central force potential $V(r)$ is expressed as—

$$L = \frac{1}{2}m\dot{r}^2 + \frac{1}{2}mr^2\dot{\theta}^2 + \frac{1}{2}mr^2 \sin^2\theta\,\dot{\phi}^2 - V(r).$$

Which one of the following is a correct statement ?

(i) Momentum conjugate to γ is conserved.

(ii) Momentum conjugate to θ is conserved.

(iii) Momentum conjugate to ϕ is conserved.

(iv) Energy is not conserved.

(e) If $V(x)$ is potential energy of a particle moving along x-direction which one of the following is a condition of stable equilibrium ?

(i) $V(x) = 0, \frac{dV}{dx} = 0$

(ii) $\frac{dV}{dx} = 0, \frac{d^2V}{dx^2} < 0$

(iii) $\frac{dV}{dx} = 0, \frac{d^2V}{dx^2} > 0$

(iv) $\frac{dV}{dx} \to \infty, \frac{d^2V}{dx^2} > 0$

(f) Which one of the following is a correct statement in special relativity ?

(i) Velocity of light depends on velocities of the observers.

(ii) If two events are simultaneous in one frame they are simultaneous in all other frames.

(iii) If two events are simultaneous in one frame they are not simultaneous other frames.

(iv) Mass of a body reduces to zero when its velocity approaches velocity of light.

(g) If momentum of a particle is $p = 2mc$, which one of the following is the correct expression for energy of the particle as per relativistic energy momentum relation ?

(i) $E = \pm 5mc^2$

(ii) $E = \pm \sqrt{5}mc^2$

(iii) $E = \pm 4mc^2$

(iv) $E = \pm 2mc^2$

(h) If $\vec{u}$ is velocity of a fluid element, which one of the following represents as incompressible fluid ?

(i) $\nabla^2 \vec{u} = 0$

(ii) $(\vec{u} \cdot \vec{\nabla})\vec{u} = 0$

(iii) $\vec{\nabla} \cdot \vec{u} = 0$

(iv) $\vec{\nabla} u^2 = 0$

2. Answer ***any six*** questions : 2×6=12

(a) Lagrangian of a simple pendulum of unit mass is given by

$$L = \frac{1}{2} l^2 \dot{\theta}^2 - gl(1 - \cos\theta).$$

Obtain the Euler-Lagrange equation.

(b) Lagrangian of a particle moving along X-direction is

$$L = \frac{1}{2} m\dot{x}^2 - \frac{1}{2} kx^4.$$

Obtain the Hamiltonian of the particle.

(c) In spherical polar coordinates Lagrangian of a free particle is given by

$$L = \frac{1}{2} m\dot{r}^2 + \frac{1}{2} mr^2\dot{\theta}^2 + \frac{1}{2} mr^2 \sin^2\theta\dot{\phi}^2.$$

Obtain the generalised momentum conjugate to ϕ when the particle moves in equatorial plane $\theta = \frac{\pi}{2}$.

(d) Lagrangian of a particle attached to a spring of spring constant K is

$L=\frac{1}{2}m\dot{x}^2-\frac{1}{2}kx^2$. Reduce Hamilton's

Canonical equation $\dot{p}_x=-\frac{\partial H}{\partial x}$ in this

case to the following form $m\ddot{x}=-kx$.

(e) A particle is displaced by an amount $x-x_0$ from its equilibrium position $x=x_0$. Obtain the Taylor expansion of potential energy $V(x)$ around the equilibrium $x=x_0$.

(f) Write down the *two* postulates of special relativity.

(g) Lorentz transformation for time is given by

$t'=\gamma\left(t-\frac{vx}{C^2}\right)$. Show that if two events

are simultaneous in one frame they are not simultaneous in the other frame.

(h) Calculate the energy equivalent to mass of the Sun, $M=2\times10^{30}kg$.

(i) Show that time derivative of velocity $(\vec{u})$ of a fluid element is

$$\frac{d\vec{u}}{dt}=\frac{\partial\vec{u}}{\partial t}+\left(\vec{u}\cdot\vec{\nabla}\right)\vec{u}.$$

(j) What is an ideal fluid ? Write down the equation of continuity.

3. Answer ***any four*** questions : 5×4=20

(a) What do you mean by stable equilibrium ? If $q_i=q_{oi}=\eta_i$ represents displacement of generalised coordinate from equilibrium (q_{0i}) expand the potential energy $V(q_1,q_2,\ldots,q_n)$ in a Taylor series about q_{0i} and obtain the potential energy matrix V_{ij}... writing the kinetic energy as $T=\frac{1}{2}m_{ij}\dot{\eta}_i\dot{\eta}_j$ and expanding the function m_{ij} in a Taylor series around q_{oi} obtain an appropriate expression for kinetic energy matrix.

1+2+2=5

(b) For a system in equilibrium derive the principle of virtual work. Apply appropriate assumption to obtain D' Alembert's principle. 2½+2½=5

(c) Lagrangian for a simple pendulum is given by $L = \frac{1}{2}ml^2\dot{\theta}^2 - mgl(1-\cos\theta)$.

Obtain the Hamiltonian and hence obtain Hamilton's Canonical equations. 3+2=5

(d) Lagrangian of a particle in cylindrical coordinate system with potential energy $V(r, \theta, z)$ is given by

$$L = \frac{1}{2}m\left(\dot{r}^2 + r^2\dot{\theta}^2 + \dot{z}^2\right) - V|r, \theta, z)$$

obtain Euler-Lagrange equations for r, θ and z.

(e) Potential energy of a particle moving along X-axis is given by

$$V(x) = -\frac{1}{2}kx^2 + \lambda x^4 \,(k, \lambda > 0).$$

Show that $x = 0, +\sqrt{k/4\lambda}$ and $-\sqrt{\frac{k}{4\lambda}}$ are equilibrium positions. Out of these three, identify the stable equilibrium positions. 2+3=5

(f) What is the inadequacy of Galilean transformation? Derive length contraction and time dilation formulae from Lorentz transformation equations. 1+2+2=5

(g) From Lorentz transformation equations of (x, t) obtain the relativistic velocity addition formula. Show that velocity of light is invariant. 4+1=5

(h) If relativistic energy and momentum are written as

$$E = \frac{mc^2}{\sqrt{1 - v^2/C^2}} \text{ and } p = \frac{mv}{\sqrt{1 - v^2/C^2}}$$

show that $\frac{E^2}{C^2} - p^2 = m^2C^2$.

Two particles, each of mass m collide head on at the speed of $V = \frac{3}{5}C$. They form a composite particle of mass M which is at rest. Use conservation of relativistic energy to show that $M = \frac{5}{2}m$. 3+2=5

4. Answer ***any two*** questions : 10×2=20

(a) Lagrangian for a particle moving under a central force potential $V(r)$ is expressed as

$$L = \frac{1}{2}m\left(\dot{r}^2 + r^2\dot{\theta}^2\right) - V(r).$$

Use Euler-Lagrange equation for θ to show that $P_\theta = mr^2\dot{\theta}$ is a conserved momentum. Show that a real velocity of the particle remains constant. Show that Euler-Lagrange equation for the coordinate r is

$$m\ddot{r} - mr\dot{\theta}^2 = f(r), \text{ where}$$

$f(r) = -\dfrac{\partial V(r)}{\partial r}$. Obtain Hamiltonian of the particle. Show that radial velocity of the particle is

$$\frac{dr}{dt} = \sqrt{\frac{2}{m}\left(E - V(r) - \frac{P_\theta^2}{2mr^2}\right)}.$$

2+2+2+2+2=10

(b) Show that Euler-Lagrange equation can be written as

$\dot{p}_i = \partial L/\partial q_i$, where p_i is the generalised momentum. If the Lagrangian is expressed as $L(q_i, \dot{q}_i, t)$ and Legendre transformation is given by

$H(q_i, p_i, t) = p_i\dot{q}_i - L(q_i, \dot{q}_i, t)$. Obtain Hamilton's Canonical equations.

2+8=10

(c) Write down Newton's second law of motion for a system of particles acted by external and internal forces. Define holonomic and non-holonomic constraints with equations and examples. A particle of mass m is falling freely under gravity vertically along Z-axis. Construct the Lagrangian. Obtain Hamilton's Canonical equation for the particle. 2+2+2+2+2=10