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3 (Sem–4/CBCS) MAT HC 1

2025

MATHEMATICS

(Honours Core)

Paper : MAT–HC–4016

(Multivariate Calculus)

Full Marks : 80

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions as directed : $1 \times 10 = 10$

(a) Let $f(x,y) = x^2y + xy^2$, if t is a real number then find $f(1-t, t)$.

(b) Evaluate $\underset{(x,y)\to(0,0)}{Lt} \dfrac{e^x \tan^{-1} y}{y}$

(c) Determine $\dfrac{\partial z}{\partial x}$, if $3x^2 + 4y^2 + 2z^2 = 5$.

(d) Define harmonic function.

(e) Find $\nabla f(x,y)$ for $f(x,y) = x^2y + y^3$

(f) Evaluate $\int_0^4 \int_0^{4-x} xy\,dy\,dx$.

(g) Define relative extrema for a function of two variables.

(h) Compute $\int_1^4 \int_{-2}^3 \int_2^5 dx\,dy\,dz$.

(i) What is the del operator?

(j) What is a vector field?

2. Answer the following questions : 2×5=10

(a) Determine f_x and f_y for

$f(x,y) = x^2 e^{x+y} \cos y$

(b) Evaluate $\int_1^2 \int_0^{\pi} x \sin y\,dy\,dx$

(c) Find the Jacobian $\dfrac{\partial(u,v)}{\partial(x,y)}$ when

$u = x - 2y,\ v = 3x - 5y$.

(d) Find the curl of the vector field

$\vec{F} = x^2yz\,\hat{i} + xy^2z\,\hat{j} + xyz^2\,\hat{k}$.

(e) Explain the difference between $\int_c f\,ds$ and $\int_c f\,dx$.

3. Answer ***any four*** questions : 5×4=20

(a) Compute the slope of the tangent line to the graph of $f(x,y) = x^2 \sin(x+y)$ at the point $P_0\left(\frac{\pi}{2}, \frac{\pi}{2}, 0\right)$.

(b) Find all relative extrema and saddle points of the function

$f(x,y) = 2x^2 + 2xy + y^2 - 2x - 2y + 5$.

(c) Evaluate

$$\iint_R x^2 e^{xy} dA;\ R: 0 \le x \le 1, 0 \le y \le 1 .$$

(d) Evaluate $\iiint_D x\,dV$, where D is the solid in the first octant bounded by the cylinder $x^2+y^2=4$ and the plane $2y+z=4$.

(e) Find the volume of the solid in the first octant that is bounded by $x^2+y^2=2y$, the half–cone $z=\sqrt{x^2+y^2}$, and the xy–plane.

(f) If $\vec{F}(x,y,z)=xy\,\hat{i}+yz\,\hat{j}+z^2\,\hat{k}$ and $\vec{G}(x,y,z)=x\,\hat{i}+y\,\hat{j}-z\,\hat{k}$ then find curl $(F\times G)$.

4. Answer ***any four*** questions : 10×4=40

(a) Let $f(x,y)=\begin{cases} xy\left(\dfrac{x^2-y^2}{x^2+y^2}\right), & (x,y)\ne(0,0) \\ 0, & (x,y)=(0,0) \end{cases}$

Show that $f_x(0,y)=-y$ and $f_x(x,0)=x$ for all x and y. Then show that $f_{xy}(0,0)=-1$ and $f_{yx}(0,0)=1$.

(b) When two resistors with resistances P and Q ohms are connected in parallel, the combine resistance is R, where

$$\frac{1}{R}=\frac{1}{P}+\frac{1}{Q}$$

If P and Q are measured at 6 and 10 ohms respectively, with error no greater than 1%, what is the maximum percentage error in the computation of R?

(c) (i) If f is differentiable and $z=u+f\left(u^2v^2\right)$. Show that

$$u\frac{\partial z}{\partial u}-v\frac{\partial z}{\partial v}=u .$$ 5

(ii) If $f(x,y)$ is a homogeneous function of degree n, show that

$$x\frac{\partial f}{\partial x}+y\frac{\partial f}{\partial y}=nf .$$ 5

(d) (i) Define directional derivative. 2

(ii) Let $f(x,y,z) = xyz$, and let $\hat{u}$ be a unit vector perpendicular to both $\vec{v} = \hat{i} - 2\hat{j} + 3\hat{k}$ and $\vec{w} = 2\hat{i} + \hat{j} - \hat{k}$. Find the directional derivative of f at $P_0(1,-1,2)$ in the direction of $\vec{u}$. 8

(e) (i) Find $div\,\vec{F}$, given that $\vec{F} = \nabla f$, where $f(x,y,z) = xy^3z^2$. 4

(ii) If $\vec{F}(x,y) = u(x,y)\hat{i} + v(x,y)\hat{j}$, Show that

Curl $\vec{F} = 0$ if and only if $\dfrac{\partial u}{\partial y} = \dfrac{\partial v}{\partial x}$. 6

(f) Let $\vec{F} = xy^2\,\hat{i} + x^2y\,\hat{j}$ and evaluate the line integral $\int_C \vec{F}.d\vec{R}$ between the points (0,0) and (2,4) along the following path :

(i) the line segment connecting the points. 4

(ii) the parabolic are $y = x^2$ connecting the points. 6

(g) Evaluate the line integral

$$\oint_C \frac{x\,dy - y\,dx}{x^2 + y^2}$$

where C is the unit circle $x^2 + y^2 = 1$ traversed once counter clockwise.

(h) Show that the vector field $\vec{F} = (e^x \sin y - y)\hat{i} + (e^x \cos y - x - 2)\hat{j}$ is conservative and then find a scalar potential function f for $\vec{F}$.