3 (Sem–6/CBCS) PHY HE 5

2024

PHYSICS

(Honours Elective)

Paper : PHY–HE–6056

(Classical Dynamics)

Full Marks : 80

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions : 1×10=10

   (a) Write the necessary and sufficient condition for force F to be conservative.

   (b) A system of 4 particles has 10 equations of constraints and requires 2 generalized coordinates. Are the constraints holonomic *or* non-holonomic ?

   (c) Write the expression for Hamiltonian of a free particle in spherical polar coordinates.

(d) State *one* advantage of Lagrangian formulation over Newtonian formulation.

(e) What is called gyro frequency? Write down its expression.

(f) Write down the relativistic form of Newton's second law of motion.

(g) What is the significance of Reynold's number?

(h) Write down the Newton's law of viscous flow in streamline motion and hence define the coefficient of viscosity.

(i) Express equation of continuity in terms of four current density vector.

(j) Write down the Lorentz transformation equations of energy and momentum.

2. Answer the following questions : 2×5=10

(a) "Magnetic field changes the velocity of a charged particle without changing its speed." Explain the statement.

(b) Show that Lagrangian and Newtonian equations of motion are equivalent.

(c) What are the different types of relativistic optical Doppler effects?

(d) State and explain postulates of special theory of relativity.

(e) A tube of radius r and length l is connected in series with another of radius $\frac{r}{2}$ and length $\frac{l}{4}$. If the pressure across the two tubes taken together is p, deduce the pressures across the tubes separately.

3. Answer ***any four*** from the following questions : 5×4=20

(a) Show that the path of a charged particle in a uniform magnetic field, in general, is a helix. Under what condition is this path reduced to a circle? 4+1=5

(b) Derive Lagrange's equations of motion for a conservative system using D' Alembert's principle.

(c) What do you mean by the element of proper time?

Using four vector expressions show that $E^2 = p^2c^2 + m_0^2c^4$, where symbols have their usual meanings. 1+4=5

(d) Write brief notes on space-like and time-like intervals.

(e) Express Lorentz transformations of space and time in four vector form.

(f) (i) Using Euler-Lagrange equation prove that 'the shortest distance between two points in a plane is a straight line'. 4

(ii) State Hamilton's principle. 1

4. Answer the following questions : 10×4=40

(a) (i) Show that Hamiltonian H is a constant of motion if the Lagrangian L is not an explicit function of time. 2

(ii) Derive Hamilton's canonical equations and use these to obtain the equation of motion of a simple pendulum. 4+4=8

Or

(b) (i) What do you mean by stable and unstable equilibria ? 2+2=4

(ii) Obtain Lagrange's equation of motion for small oscillations of a system in the neighbourhood of stable equilibrium. 4

(iii) Write the principle of virtual work in terms of independent generalized coordinates. 2

(c) (i) The equation of the orbit of a particle under the action of a central force is $r = 2a\cos\theta$.

Show that the force F is inversely proportional to r^5. 4

(ii) Prove the conservation of energy of a particle directly from its equation of motion in a central force field. 4

(iii) The nature of orbit is determined by the value of its eccentricity

$$\varepsilon = \sqrt{1 + \frac{2EL^2}{\mu k^2}}$$

where symbols have their usual meanings.

Mention the various special cases depending upon the values of E and ε. 2

Or

(d) (i) A proton with initial velocity of $5 \times 10^6\ ms^{-1}$ passes through an electric field (transverse) of $200\ volt/cm$. Calculate the transverse deflection in travelling a distance of 1 *m*. 3

(ii) Obtain equations of motion of a system of coupled simple pendulums by setting Lagrangian of the entire system. 7

(e) (i) Explain the concept of twin paradox with the help of space-time diagram. 5

(ii) Draw a neat diagram of light cones indicating past and future. Show the world lines in it.

Can the tangent to the world line of a massive particle at a point has an angle equal to or more than 45° ? Explain. 3+2=5

Or

(f) (i) What are called cyclic or ignorable coordinates ? If a system undergoes translatory motion along a cyclic generalized coordinate q_k, will the Lagrangian of the system be affected ? 2

(ii) Obtain the Lagrange's equation of motion for an electrical circuit comprising an inductance *L* and capacitance *C*. The capacitor is charged to *q* coulombs and the current flowing in the circuit is *i* amperes. 3

(iii) Show that Lorentz transformations of space and time can be regarded as transformations due to rotation of axes in the four-dimensional Minkowski space. 5

(g) (i) State length centraction and time dilation. How are the phenomenon of length contraction and time dilation interpreted on space-time diagram ? 2+(3+3)=8

(ii) Calculate the velocity which 1 *amu* of mass will have, if it had a kinetic energy 3 times the rest mass energy. 2

Or

(h) *(i)* The rate of a liquid through a capillary tube is $V = \frac{P\pi r^4}{8\eta l}$ with usual notations. Deduce the relation stating clearly the conditions under which it holds. 6

(ii) Discuss the corrections to be applied to Poiseuille's equation. 4