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3 (Sem-3/CBCS) STA HC 1

2023

STATISTICS

(Honours Core)

Paper : STA-HC-3016

(Sampling Distributions)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions as directed :

1×7=7

(a) The standard error of the sampling distribution of the statistic (\bar{x}) is

(Choose the correct option)

(i) $\sigma^2 \sqrt{2/n}$

(ii) σ/\sqrt{n}

(iii) $\sqrt{\sigma^2/2n}$

(iv) None of the above

Contd.

(b) The cumulative distribution function of the largest order statistic $X_{(n)}$ is given by _____. (Fill in the blank)

(c) For large n if $X \sim N(nP, nPQ)$, then

$$Z = \frac{X - nP}{\sqrt{nPQ}} \text{ follows}$$

(Choose the correct option)

(i) $N(0, \sigma^2)$

(ii) $N(\mu, \sigma^2)$

(iii) $N(0, 1)$

(iv) $N(\mu, \sigma)$

(d) If $X_i, i = 1, 2, \dots, n$ are n independent normal variates with mean (μ_i) and SD

(σ_i) , then $\sum_{i=1}^n \left(\frac{X_i - \mu}{\sigma} \right)^2$ is a chi-square variate with n d.f.

(Write true or false)

(e) The probability of type I error is called _____. (Fill in the blank)

(f) State the pdf of Fisher's t -distribution.

(g) Equality of two population variances can be tested by

(Choose the correct option)

(i) t -test

(ii) F -test

(iii) Both (i) and (ii)

(iv) None of the above

2. Answer the following questions : $2 \times 4 = 8$

(a) Derive the cumulative distribution function of $X_{(1)}$.

(b) Write any two applications of chi-square statistic.

(c) Explain one tailed and two tailed tests.

(d) Write the assumptions for students t-test.

3. Answer **any three** questions from the following : $5 \times 3 = 15$

(a) Explain in brief the test used for testing the difference between two proportions for large samples.

(b) Find the joint distribution of r^{th} and s^{th} order statistics ($r < s$) in taking random sample from a continuous distribution.

(c) Derive cumulant generating function (c.g.f.) of chi-square distribution. Also find its mean and variance using c.g.f.

(d) Define F statistic. Write down the p.d.f. of Snedecor's F distribution. Derive the mode of F distribution. $1+1+3=5$

(e) In $F(n_1, n_2)$ distribution and if $n_2 \rightarrow \infty$, then prove that $n_1 F$ follows chi-square distribution with n_1 d.f.

Answer either 4. (a) or 4. (b) :

4. (a) Obtain the distribution of sample median in case of order statistics. 10

(b) (i) Let X_1 and X_2 be two independent normal variates with the same normal distribution $N(\mu, \sigma^2)$. Obtain the distribution of

$$Y = \frac{X_1 + X_2 - 2\mu}{\sqrt{|X_1 - X_2|^2}} \quad 5$$

(ii) If X is t -distributed with K degrees of freedom, show that $\frac{1}{1+(X^2/K)}$ has a beta distribution. 5

Answer either 5. (a) or 5. (b) :

5. (a) If X_1 and X_2 are two independent chi-square variate with n_1 and n_2 d.f. respectively, then show that X_1/X_2 is

a $\beta_2(n_1/2, n_2/2)$ variate. 10

(b) (i) Describe the steps in detail for testing a statistical hypothesis. 5

(ii) For t -distribution with n d.f., derive the mean deviation about mean. 5

Answer either 6. (a) or 6. (b) :

6. (a) (i) Derive the probability density function of student's t . 7

(ii) Comment on the graph of t -distribution. 3

(b) Write three applications of F distribution.

Let X_1 and X_2 be a random sample of size 2 from $N(0, 1)$ and Y_1 and Y_2 be a random sample of size 2 from $N(1, 1)$ and let Y_i 's be independent of X_i 's. Find the distribution of the following :

3+7=10

(i)
$$\frac{(X_1 + X_2)^2}{(X_2 - X_1)^2}$$

(ii)
$$\frac{(Y_1 + Y_2 - 2)^2}{(X_2 - X_1)^2}$$