Total number of printed pages–7

3 (Sem–5/CBCS) STA HC1

2024

STATISTICS

(Honours Core)

Paper : STA–HC–5016

(Stochastic Processes and Queuing Theory)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions : 1×7=7

   (a) What is the Markovian property of a stochastic process ?

   (b) What is absorbing barrier ?

   (c) The mean of the random variable X in terms of the probability generating function $p(s)$ of X is given by $E(X) = p'(s)$. *(State True **or** False)*

Contd.

(d) The difference of two independent Poisson processes is not a Poisson process. *(State True **or** False)*

(e) The probability generating function of the sum of two independent random variables X and Y is the ________ of the generating function of X and Y. *(Fill in the blank)*

(f) Define stochastic process.

(g) What is meant by steady state distribution ?

2. Answer the following questions : 2×4=8

(a) Mention *two* uses of stochastic process.

(b) Classify the Markov chain with transition probability.

$$\begin{array}{c} \\ 0 \\ 1 \\ 2 \end{array}\begin{array}{c} \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{bmatrix} 0 & \frac{1}{3} & \frac{2}{3} \\ \frac{2}{3} & 0 & \frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} & 0 \end{bmatrix} \end{array}$$

(c) Is poisson process a stationary process ?

(d) Define bivariate probability generating function of a pair of random variable X and Y.

3. Answer ***any three*** of the following questions : 5×3=15

(a) Prove that the sum of two independent Poisson process is a Poisson process.

(b) Prove that the interval between two successive occurrences of a Poisson process having parameter λ has a negative exponential distribution with mean $1/\lambda$.

(c) Write a note on Order of Markov chain.

(d) The transition probability matrix of a Markaov chain with three states 0, 1, 2 is

$$P = \begin{bmatrix} 3/4 & 1/4 & 0 \\ 1/4 & 1/2 & 1/4 \\ 0 & 3/4 & 1/4 \end{bmatrix}$$

Draw the transition probability graph and hence verify whether the process is transient or not.

(e) Derive the probability distribution of number of customer in M/M/1 queuing model with finite system capacity.

4. Answer **either** (a) **or** (b) :

(a) (i) Write a note on "graphical representation of Markov chain." 5

(ii) Derive Chapman-Kolmogorov equation. 5

(b) (i) Define Poisson process ? Describe the postulates for Poisson process. 5

(ii) Prove that for a Poisson process $N(t)$, as $t \to \infty$

$$Pr\left\{\left|\frac{N(t)}{t} - \lambda\right| \geq \varepsilon\right\} \to 0$$

where $\varepsilon > 0$ is a preassigned number. 5

5. Answer **either** (a) **or** (b) :

(a) (i) Define Queuing system. What is "steady state" of a Queuing system ? 5

(ii) If $\{N(t)\}$ is a Poisson process and $s < t$, then prove that

$$Pr\{N(s) = k/N(t) = n\} = \binom{n}{k}\left(s/t\right)^k \left(1 - s/t\right)^{n-k}$$

5

(b) (i) Write a note on "Gennalization of Independent Bernoulli trials". 5

(ii) Write a note on "specification of stochastic processes." 5

6. Answer **either** (a) **or** (b) :

(a) Under the postulates for Poisson process, prove that $N(t)$ follows Poisson distribution with mean λt. i.e. $p_n(t)$ is given by the Poisson law.

$$p_n(t) = \frac{e^{-\lambda t}(\lambda t)^n}{n!}, \ n = 0,1,2,.....$$

10

(b) Cars and bikes arrive at a shopping mall according to Poisson distribution at the rate of 2 cars every 5 minutes. The parking spare in front of the shopping mall can accommodate atmost 10 cars/ bikes includingthe one being served. If the parking lot is full, the cars/bikes are turned away. There is only one service counter. The service time per customer is exponential with a mean of 1.5 minutes. Find

(i) The Queing model.

(ii) The arrival rate and service rate.

(iii) The traffic intensity and give interpretation.

(iv) The probability that there are no customer in the shopping mall.

(v) The probability that an arriving customer is turned away.

2+2+2+2+2=10

1500