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3 (Sem–3/CBCS) MAT HC 3

2023

MATHEMATICS

(Honours Core)

Paper : MAT–HC–3036

(Analytical Geometry)

Full Marks : 80

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer ***all*** the questions : 1×10=10

(a) When the origin is shifted to a point on the *x*-axis without changing the direction of the axes, the equation of the line $2x+3y-6=0$ takes the form $lx+my=0$. What is the new origin ?

(b) Find the centre of the ellipse $2x^2+3y^2-4x+5y+4=0$.

(c) Find the angle between the lines represented by the equation

$x^2 + xy - 6y^2 = 0$.

(d) Transform the equation $\frac{1}{r} = 1 + \cos\theta$ into cartesian form.

(e) Find the equation of the tangent to the conic $y^2 - xy - 2x^2 - 5y + x - 6 = 0$ at the point $(1, -1)$.

(f) Express the non-symmetric form of equation of a line $\frac{y}{p} + \frac{z}{c} = 1$, $x = 0$ in symmetric form.

(g) Write down the standard form of equation of a system of coaxial spheres.

(h) Write down the equation of a cone whose vertex is origin and the guiding curve is $ax^2 + by^2 + cz^2 = 1$,

$lx + my + nz = p$.

(i) Define a right circular cylinder.

(c) Find the angle between the lines represented by the equation

$x^2 + xy - 6y^2 = 0$.

(d) Transform the equation $\frac{1}{r} = 1 + \cos\theta$ into cartesian form.

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(g) Write down the standard form of equation of a system of coaxial spheres.

(h) Write down the equation of a cone whose vertex is origin and the guiding curve is $ax^2 + by^2 + cz^2 = 1$,

$lx + my + nz = p$.

(i) Define a right circular cylinder.

(j) Find the equation of the tangent plane to the conicoid

$$ax^2 + by^2 + cz^2 = 1$$

at the point (α, β, γ) on it.

2. Answer **all** the questions : 2×5=10

(a) If $(at^2, 2at)$ is the one end of a focal chord of the parabola $y^2 = 4ax$, find the other end.

(b) Show that the equation of the lines through the origin, each of which makes an angle α to the line $y = x$ is

$x^2 - 2xy\sec 2\alpha + y^2 = 0$.

(c) Find the point where the line

$$\frac{x-1}{2} = \frac{y-2}{-3} = \frac{z+3}{4}$$

meets the plane $x + y + z = 3$.

(d) Find the equation of the sphere passing the points
(0, 0, 0), (a, 0, 0), (0, b, 0), (0, 0, c)

(e) Find the equation of the plane which cuts the surface

$2x^2 - 3y^2 + 5z^2 = 1$ in a conic whose centre is (1, 2, 3).

3. Answer **any four** questions : 5×4=20

(a) Show that the equation of the tangent to the conic $\frac{l}{r} = 1 + e\cos\theta$ at the point whose vertical angle is α is given by

$$\frac{l}{r} = e\cos\theta + \cos(\theta - \alpha).$$

(b) Prove that the line $lx + my = n$ is a normal to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, if

$$\frac{a^2}{l^2} + \frac{b^2}{m^2} = \frac{\left(a^2 - b^2\right)}{n^2}.$$

(c) Find the asymptotes of the hyperbola $2x^2 - 3xy - 2y^2 + 3x + y + 8 = 0$ and derive the equations of the principal axes.

(d) Prove that the lines

$$\frac{x+5}{3}=\frac{y+4}{1}=\frac{z-7}{-2} \text{ and}$$

$3x+2y+z-2=0=x-3y+2z-13$ are coplanar. Find the equation of the plane in which they lie.

(e) The section of a cone whose guiding curve is the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1, z=0$ by the plane $x=0$, is a rectangular hyperbola. Prove that the locus of the vertex is $\frac{x^2}{a^2}+\frac{y^2+z^2}{b^2}=1$.

(f) Find the centre and the radius of the circle

$$x^2+y^2+z^2-8x+4y+8z-45=0,$$
$$x-2y+2z=3.$$

Answer **either** (a) **or** (b) from the following questions : 10×4=40

4. (a) (i) Find the point of intersection of the lines represented by the equation

$$ax^2+2hxy+by^2+2gx+2fy+c=0$$

(ii) Find the equation of the polar of the point (2, 3) with respect to the conic $x^2 + 3xy + 4y^2 - 5x + 3 = 0$.

5+5=10

(b) (i) Prove that the straight line $y = mx + c$ touches the parabola $y^2 = 4a(x+a)$ if $c = ma + \frac{a}{m}$.

(ii) Find the asymptotes of the hyperbola $xy + ax + by = 0$.

5+5=10

5. (a) Discuss the nature of the conic represented by

$$9x^2 - 24xy + 16y^2 - 18x - 101y + 19 = 0$$

and reduce it to canonical form.

(b) (i) Prove that the sum of the reciprocals of two perpendicular focal chords of a conic is constant.

(ii) Show that the semi-latus rectum of a conic is the harmonic mean between the segments of a focal chord.

5+5=10

6. *(a)* *(i)* A variable plane makes intercepts on the co-ordinate axes, the sum of whose squares is a constant and is equal to k^2. Prove that the locus of the foot of the perpendicular from the origin to the plane is

$$\left(x^2+y^2+z^2\right)^2\left(x^{-2}+y^{-2}+z^{-2}\right)=k^2$$

(ii) Two spheres of radii r_1 and r_2 intersect orthogonally. Prove that the radius of the common circle is

$$\frac{r_1 r_2}{\sqrt{r_1^2+r_2^2}}.$$

5+5=10

(b) Show that the shortest distance between any two opposite edges of the tetrahedron formed by the planes $y+z=0,\ z+x=0,\ x+y=0,\ x+y+z=a$ is $\frac{2a}{\sqrt{6}}$ and that the three lines of shortest distance intersect at the point $x=y=z=-a$.

7. *(a)* *(i)* Define reciprocal cone. Show that the cones $ax^2 + by^2 + cz^2 = 0$ and $\frac{x^2}{a} + \frac{y^2}{b} + \frac{z^2}{c} = 0$ are reciprocal.

(ii) Find the equation of the right circular cylinder whose guiding curve is $x^2 + y^2 + z^2 = 9$, $x - y + z = 3$.

5+5=10

(b) *(i)* Find the equation of the director sphere to the ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1.$$

(ii) Show that from any point six normals can be drawn to a conicoid $ax^2 + by^2 + cz^2 = 1$.

5+5=10
