2023

PHYSICS

(Honours Core)

Paper : PHY–HC–3016

(Mathematical Physics–II)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions : 1×7=7

(a) Define the ordinary point of a second order differential equation.

(b) Show that $P_0(x)=1$.

(c) Write the Laplace equation in spherical polar co-ordinate system.

(d) A partial differential equation has

(i) one independent variable

(ii) more than one dependent variable

(iii) two or more independent variables

(iv) no independent variable.

(Choose the correct option)

(e) If A and B are two square matrices of order n, show that

$$Trace\,(A+B) = Trace\,A + Trace\,B$$

(f) Which one of the following is the value of $\Gamma\left(\frac{1}{2}\right)$?

(i) $\sqrt{\pi/2}$

(ii) $\sqrt{\pi}$

(iii) π

(iv) $\sqrt{\pi}/2$

(g) Define self adjoint of a matrix.

2. Answer the following questions : 2×4=8

(a) Check the behaviour of point $x=0$ for the differential equation

$$\frac{d^2y}{dx^2} - \frac{6}{x}y = 0$$

(b) If $\int_{-1}^{+1} P_n(x)\,dx = 2$, find the value of n.

(c) Express the Fourier series in complex form.

(d) Verify the matrix

$$A = \begin{bmatrix} \cos\theta & i\sin\theta \\ i\sin\theta & \cos\theta \end{bmatrix}$$ is a unitary

matrix.

3. Answer **any three** of the following questions : 5×3=15

(a) Find the power series solution of the following differential equation :

$$\left(1-x^2\right)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$$

about $x = 0$.

(b) Define Gamma function. Show that

$$\int_0^{\infty} e^{-x^2}dx = \frac{\sqrt{\pi}}{2}$$ 1+4=5

(c) Establish the following recurrence formula for Legendre polynomial $P_n(x)$

$$nP_n(x) = (2n-1)xP_{n-1}(x) - (n-1)P_{n-2}(x)$$

(d) Show that the Fourier expansion of the function $f(x) = x$, $0 < x < 2\pi$ is

$$x = \pi - 2\left[\sin x + \frac{1}{2}\sin 2x + \frac{1}{3}\sin 3x + \ldots\right]$$

(e) What is eigenvalue of a matrix? Show that the eigenvalues of Hermitian matrix are real. 1+4=5

4. Answer ***any three*** of the following questions : 10×3=30

(a) (i) If $P_n(x)$ be the polynomial of Legendre's differential equation, show that

$$P_n(x) = \frac{1}{2^n . n!} \cdot \frac{d^n}{dx^n}(x^2-1)^n$$ 6

(ii) Prove that 4

$$\int_{-1}^{+1} xP_n(x)P_{n-1}(x)\,dx = \frac{2n}{4n^2-1}$$

(b) (i) What is Beta function ? Show that

(a) $\beta(1,1)=1$

(b) $\beta(m,n)=\beta(n,m)$ 1+1+3=5

(ii) If $A=\begin{bmatrix}1 & 2 & 2\\ 2 & 1 & 2\\ 2 & 2 & 1\end{bmatrix}$, show that

$A^2-4A-5I=0$ where I and 0 are the unit matrix and the null matrix of order 3 respectively. Also use this result to find A^{-1}. 3+2=5

(c) Find the Fourier series expansion of

$$f(x)=\begin{cases}-\pi, & -\pi<x<0\\ x, & 0<x<\pi\end{cases}$$

Also show that

$$\frac{\pi^2}{8}=\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\cdots$$ 6+4=10

(d) (i) Diagonalize the following matrix

$$A=\begin{bmatrix}1 & 0 & 1\\ 1 & 1 & 0\\ 1 & 0 & 1\end{bmatrix}$$ 5

(ii) Show that the generating function for Hermite polynomial $H_n(x)$, for integral n and real value of n is given by

$$e^{2xt-t^2} = \sum_{n=0}^{\infty} \frac{t^n}{n!} H_n(x)$$ 5

(e) (i) Write the one dimensional diffusion equation (heat flow equation) and find the general solution of the same by the method of separation of variable. 1+7=8

(ii) For the Pauli spin matrices

$$\sigma_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad \sigma_2 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \text{ and}$$

$$\sigma_3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \text{ show that}$$

$$[\sigma_1, \sigma_2] = 2i\sigma_3$$ 2

(f) (i) Write the Orthogonality conditions of *sine* and *cosine* functions. 1½+1½=3

(ii) A square wave function is represented as

$$f(x) = \begin{cases} 0, & \text{for } -\pi < x < 0 \\ h, & \text{for } 0 \le x < \pi \end{cases}$$

Draw the graphical representation of the wave function and expand the same in Fourier series.

1+6=7
