

Total number of printed pages-20

3 (Sem-3/CBCS) MAT HG 1/2/RC

2023

MATHEMATICS

(Honours Generic/Regular)

Answer the Questions from any one Option.

OPTION-A

Paper : MAT-HG-3016 / MAT-RC-3016

(Differential Equation)

OPTION-B

Paper : MAT-HG-3026

(Linear Programming)

Full Marks : 80

Time : Three hours

***The figures in the margin indicate
full marks for the questions.***

Contd.

OPTION-A

Paper : MAT-HG-3016 / MAT-RC-3016

(*Differential Equation*)

Answer **either** in English **or** in Assamese.

1. Answer the following questions: $1 \times 10 = 10$

তলত দিয়া প্ৰশ্নবোৰৰ উত্তৰ দিয়া :

(a) Define order and degree of an ordinary differential equation.

সাধাৰণ অৱকল সমীকৰণৰ ক্ৰম আৰু ঘাতৰ সংজ্ঞা লিখা।

(b) What do you mean by an ordinary differential equation? Give *one* example.

সাধাৰণ অৱকল সমীকৰণ বুলিলে কি বুজা? এটা উদাহৰণ দিয়া।

(c) Define exact differential equation.

যথার্থ অৱকল সমীকৰণৰ সংজ্ঞা লিখা।

(d) Obtain the differential equation of family of parabolas given by $y^2 = 4ax$.

$y^2 = 4ax$ অধিবৃত্তৰ পৰিয়ালটোৰ অৱকল সমীকৰণটো গঠন কৰা।

- (e) Write the condition of exactness of an ordinary differential equation.

এটা সাধাৰণ অৱকল সমীকৰণৰ যথার্থতাৰ চৰ্ত লিখা।

- (f) Find the integrating factor of

$$\frac{dy}{dx} + \frac{y}{x} = \cos x.$$

$$\frac{dy}{dx} + \frac{y}{x} = \cos x, \text{ ৰ অনুকলন গুণক নিৰ্ণয় কৰা।}$$

- (g) Define orthogonal trajectory of a family of curve.

এটা বক্ৰ পৰিয়ালৰ লাম্বিক প্ৰক্ষেপপথৰ সংজ্ঞা লিখা।

- (h) Write the complementary function of

$$(D^2 + 4)y = x^2.$$

$(D^2 + 4)y = x^2$ অৱকল সমীকৰণটোৰ পৰিপূৰক ফলনটো লিখা।

- (i) Write the general form of a linear differential equation of n^{th} order.

এটা n মাত্ৰাৰ ৰৈখিক অৱকল সমীকৰণৰ সাধাৰণ ৰূপটো লিখা।

(i) If $y_1 = \sin 2x$ and $y_2 = \cos 2x$, then find the Wronskian of $y_1(x)$ and $y_2(x)$.

যদি $y_1 = \sin 2x$ আৰু $y_2 = \cos 2x$, তেন্তে $y_1(x)$ আৰু $y_2(x)$ ৰ Wronskian নিৰ্ণয় কৰা।

2. Answer the following questions : $2 \times 5 = 10$

তলত দিয়া প্ৰশ্নবোৰৰ উত্তৰ দিয়া :

(a) Determine the particular integral of the differential equation

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} + 1 = \sin 2x.$$

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} + 1 = \sin 2x \text{ অৱকল সমীকৰণটোৰ}$$

বিশেষ অনুকলন নিৰ্ণয় কৰা।

(b) Derive the orthogonal trajectory of $xy = a^2$.

$xy = a^2$, ৰ লাম্বিক প্ৰক্ষেপপথ নিৰ্ণয় কৰা।

- (c) Find the integrating factor of the differential equation

$$(x^2y - 2xy^2)dx - (x^3 - 3x^2y)dy = 0$$

$$(x^2y - 2xy^2)dx - (x^3 - 3x^2y)dy = 0$$

অৱকল সমীকৰণটোৰ অনুকলন গুণক নিৰ্ণয় কৰা।

(d) Solve: $\frac{dx}{y^2} = \frac{dy}{x^2} = \frac{dz}{x^2y^2z^2}$

সমাধান কৰা: $\frac{dx}{y^2} = \frac{dy}{x^2} = \frac{dz}{x^2y^2z^2}$

(e) Solve: $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 13y = 0$

সমাধান কৰা: $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 13y = 0$

3. Answer the following: **(any four)** $5 \times 4 = 20$

তলত দিয়া প্ৰশ্নবোৰৰ উত্তৰ দিয়া: (যিকোনো চাৰিটা)

(a) Solve: $x^2 \frac{d^2y}{dx^2} - 3x \frac{dy}{dx} + 4y = 2x^2$

সমাধান কৰা: $x^2 \frac{d^2y}{dx^2} - 3x \frac{dy}{dx} + 4y = 2x^2$

- (b) Find the orthogonal trajectories of the series of hypocycloid $x^{2/3} + y^{2/3} = a^{2/3}$.

$x^{2/3} + y^{2/3} = a^{2/3}$, পৰিয়ালটোৰ লাম্বিক প্ৰক্ষেপপথ নিৰ্ণয় কৰা।

- (c) Solve the simultaneous linear differential equations $\frac{dx}{dt} = -py$ and

$\frac{dy}{dt} = px$ and show that the point (x, y) lies on a circle.

$\frac{dx}{dt} = -py$ আৰু $\frac{dy}{dt} = px$; অৱকল সমীকৰণটো

সমাধান কৰা আৰু দেখুওৱা যে (x, y) বিন্দুটো এটা বৃত্তত থাকিব।

- (d) Solve by reducing to exact differential equation

$$xydx + (2x^2 + 3y^2 - 20)dy = 0$$

$xydx + (2x^2 + 3y^2 - 20)dy = 0$ সমীকৰণক যথার্থ অৱকল সমীকৰণলৈ সমানীত কৰি সমাধান কৰা।

(e) Solve the Bernoulli's equation :

$$x \frac{dy}{dx} + y = y^2 \log x$$

বোৰ্ণৌলীৰ সমীকৰণটো সমাধান কৰা :

$$x \frac{dy}{dx} + y = y^2 \log x$$

(f) Solve $x^2 \frac{d^2y}{dx^2} - 3x \frac{dy}{dx} + 4y = 0$, given

that $y = x^2$ is one of the solution.

$$x^2 \frac{d^2y}{dx^2} - 3x \frac{dy}{dx} + 4y = 0 \text{ অৱকল সমীকৰণটো}$$

সমাধান কৰা, য'ত সমীকৰণটোৰ এটা সমাধান $y = x^2$.

4. Answer the following : **(any four)** $10 \times 4 = 40$

তলত দিয়া প্ৰশ্নবোৰৰ উত্তৰ দিয়া : (যিকোনো চাৰিটা)

(a) Solve by the method of variation of

$$\text{parameter : } \frac{d^2y}{dx^2} - y = \frac{2}{1+e^x}$$

প্ৰাচল বিচৰণ পদ্ধতিৰে সমাধান কৰা :

$$\frac{d^2y}{dx^2} - y = \frac{2}{1+e^x}$$

(b) Solve : $\frac{d^4 y}{dx^4} - y = x \sin x$

সমাধান করা : $\frac{d^4 y}{dx^4} - y = x \sin x$

(c) Solve : $\frac{dx}{dt} + \frac{dy}{dt} + 2x + y = 0$

$$\frac{dy}{dt} + 5x + 3y = 0$$

সমাধান করা : $\frac{dx}{dt} + \frac{dy}{dt} + 2x + y = 0$

$$\frac{dy}{dt} + 5x + 3y = 0$$

(d) Solve the exact differential equation :

$$x^2 \frac{d^2 y}{dx^2} + 3x \frac{dy}{dx} + y = \frac{1}{(1-x)^2}$$

যথার্থ অরকল সমীকরণটো সমাধান করা :

$$x^2 \frac{d^2 y}{dx^2} + 3x \frac{dy}{dx} + y = \frac{1}{(1-x)^2}$$

(e) Solve by reducing to normal form

$$\frac{d^2y}{dx^2} - 4x \frac{dy}{dx} + (4x^2 - 1)y = -3e^{x^2} \sin 2x$$

নৰ্মাল ৰূপলৈ সমানীত কৰি সমাধান কৰা :

$$\frac{d^2y}{dx^2} - 4x \frac{dy}{dx} + (4x^2 - 1)y = -3e^{x^2} \sin 2x$$

(f) Show that the term $\frac{1}{x(x^2 - y^2)}$ is an

integrating factor of the differential equation $(x^2 + y^2)dx - 2xy dy = 0$ and hence solve it.

দেখুওৱা যে $(x^2 + y^2)dx - 2xy dy = 0$

সমীকৰণৰ এটা অনুকলন গুণক $\frac{1}{x(x^2 - y^2)}$ আৰু

সমাধান কৰা।

(g) Solve the equation, $4y = x^2 + p^2$, where

$$p \equiv \frac{dy}{dx}$$

সমাধান কৰা : $4y = x^2 + p^2$, যত $p \equiv \frac{dy}{dx}$

(h) Discuss the method of solving a Bernoulli's equation of the form

$$\frac{dy}{dx} + Py = Qy^n; \text{ where } P \text{ and } Q \text{ are}$$

constants as function of x .

এটা $\frac{dy}{dx} + Py = Qy^n$ ৰূপৰ বাৰ্নৌলীৰ সমীকৰণ

সমাধান কৰাৰ পদ্ধতি আলোচনা কৰা, য'ত P আৰু Q হৈছে ধ্রুবক বা x ৰ ফলন।

OPTION-B

Paper : MAT-HG-3026

(Linear Programming)

1. Answer the following questions : (Choose the correct answer) 1×10=10

(a) A basic feasible solution whose variables are

- (i) degenerate
- (ii) non-degenerate
- (iii) non-negative
- (iv) None of the above

(b) The inequality constraints of an LPP can be converted into equation by introducing

- (i) negative variables
- (ii) non-degenerate B.F.
- (iii) slack and surplus variables
- (iv) None of the above

(c) A solution of an LPP, which optimize the objective function is called

- (i) basic solution
- (ii) basic feasible solution
- (iii) optimal solution
- (iv) None of the above

(d) Given a system of m simultaneous linear equations in n unknowns ($m < n$) the number of basic variables will be

- (i) m
- (ii) n
- (iii) $n - m$
- (iv) $n + m$

(e) A simplex in n -dimension is a convex polyhedron having

- (i) $n - 1$ vertices
- (ii) n vertices
- (iii) $n + 1$ vertices
- (iv) None of the above

(f) At any iteration of the usual simplex method, if there is at least one basic variable in the basis at zero level and all $z_j - c_j \geq 0$ the current solution is

- (i) infeasible
- (ii) unbounded
- (iii) non-degenerate
- (iv) degenerate

(z_j, c_j having usual meaning)

(g) Let $X = \{x_1, x_2\} \subset \mathbb{R}^2$. Then the convex hull $C(X)$ of X is

- (i) $\{\lambda x_1 + (1 - \lambda)x_2 : \lambda \geq 1\}$
- (ii) $\{\lambda x_1 + (1 - \lambda)x_2 : \lambda \leq 0\}$
- (iii) $\{\lambda x_1 + (1 - \lambda)x_2 : 0 < \lambda < 1\}$
- (iv) None of the above

(h) For given linear programming problem, if z is an objective function

- (i) $\text{Max } z = - \text{Min } z$
- (ii) $\text{Max } z = \text{Min } (-z)$
- (iii) $\text{Max } (-z) = \text{Max } z$
- (iv) None of above

- (i) The set $\{(x_1, x_2): x_1^2 + x_2^2 \leq 1\}$ is a
- (i) open set
 - (ii) closed set
 - (iii) neither open nor closed
 - (iv) open and closed both
- (j) In linear programming problem
- (i) objective function, constraints and variables are all linear
 - (ii) only objective function to be linear
 - (iii) only constraints are to be linear
 - (iv) only variables are to be linear

2. Answer the following :

$$2 \times 5 = 10$$

- (a) A hyperplane is given by the equation $3x_1 + 2x_2 + 4x_3 + 7x_4 = 8$, find in which half space do the point $(-6, 1, 7, 2)$ lie.
- (b) Prove that $x_1 = 2$, $x_2 = -1$ and $x_3 = 0$ is a solution but not a basic solution to the system of equations

$$3x_1 - 2x_2 + x_3 = 8$$

$$9x_1 - 6x_2 + 4x_3 = 24$$

- (c) Write the dual of the following primal problem :

$$\text{Minimize } Z = 3x_1 + 5x_2$$

$$\text{subject to } 3x_1 + 5x_2 = 12$$

$$4x_1 + 2x_2 = 10$$

$$\text{with } x_1, x_2 \geq 0$$

- (d) In a two-person Zero-sum game, the pay-off matrix is given by

		B		
		I	II	III
A	I	6	8	6
	II	4	12	2

Find its saddle points.

- (e) Show that the linear function

$Z = C X, X \in \mathbb{R}^n, C \in \mathbb{R}$ is a convex function.

3. Answer **any four** of the following : $5 \times 4 = 20$

(a) Solve graphically the following LPP :

$$\text{Max. } Z = 5x_1 + 7x_2$$

$$\text{subject to } x_1 + x_2 \leq 4$$

$$3x_1 + 8x_2 \leq 24$$

$$10x_1 + 7x_2 \leq 35$$

$$x_1, x_2 \geq 0$$

(b) Find all basic feasible solutions of the system of equations

$$x_1 + 2x_2 + 3x_3 + 4x_4 = 7$$

$$2x_1 + x_2 + x_3 + 2x_4 = 3$$

(c) Prove that the set of all convex combinations of a finite number of points $x_1, x_2, x_3, \dots, x_n$ is a convex set.

(d) Prove that the dual of a dual is a Primal problem itself.

- (e) Solve the following transportation problem using North-West corner method whose cost matrix is given below :

Source	D_1	D_2	D_3	D_4	Supply
S_1	7	10	14	8	30
S_2	7	11	12	6	40
S_3	5	8	15	9	30
Demand	20	20	25	35	

- (f) The pay-off matrix of a game is given below. Find the solution of the game to A and B.

		B				
		I	II	III	IV	V
A	I	-2	0	0	5	3
	II	3	2	1	2	2
	III	-4	-3	0	-2	6
	IV	5	3	-4	2	-6

4. Answer **any four** questions : $10 \times 4 = 40$

(a) Old hens can be bought for Rs. 2 each but young ones cost Rs. 5 each. The old hens lay 3 eggs per week and the young ones 5 eggs per week, each being worth 30 paise. A hen costs Re. 1 per week to feed. If I have only Rs. 80 to spend for hens, how many of each kind shall I buy to give a profit of more than Rs. 6 per week, assuming that I can not house more than 20 hens? Formulate the LPP and solve by graphical method.

(b) Prove that if either the primal or the dual problem of an LPP has a finite optimal solution, then the other problem also has a finite optimal solution. Furthermore, the optimal values of the objective function in both the problems are the same, i.e.

$$\text{Max } Z_x = \text{Max } Z_w$$

(c) Solve the following assignment problem :

Projects

	A	B	C	D
I	12	10	10	8
II	14	Not suitable	15	11
III	6	10	16	4
IV	8	10	9	7

(d) Use simplex method to solve the LPP

$$\text{Max } Z = 4x + 10y$$

subject to the constraints

$$2x + y \leq 50$$

$$2x + 5y \leq 100$$

$$2x + 3y \leq 90$$

$$x, y \geq 0$$

(e) Use the two-phase simplex method to solve $\text{Max } Z = 5x_1 - 4x_2 + 3x_3$

subject to the constraints

$$2x_1 + x_2 - 6x_3 = 20$$

$$6x_1 + 5x_2 + 10x_3 \leq 76$$

$$8x_1 - 3x_2 + 6x_3 \leq 50$$

$$x_1, x_2, x_3 \geq 0$$

(f) Solve the game whose pay-off matrix is

$$\begin{bmatrix} -1 & -2 & 8 \\ 7 & 5 & -1 \\ 6 & 0 & 12 \end{bmatrix}$$

(g) If in an assignment problem, a constant is added or subtracted to every element of a row (or column) of the cost matrix $[c_{ij}]$, then prove that an assignment which minimizes the total cost for one matrix, also minimizes the total cost for the other matrix.

(h) (i) What is game theory? 2

(ii) Describe a two-person zero-sum game. Also mention *any two* basic assumptions in it. 4

(iii) Explain the following terms

Optimal strategy, Pay-off matrix.

2+2=4

Total number of printed pages-19

3 (Sem-3/CBCS) CSC/TCA/TIT (HG/RC)

2023

**COMPUTER SCIENCE/
COMPUTER APPLICATION / INFORMATION
TECHNOLOGY**

(Honours Generic/Regular)

Answer the Questions from any one Option.

OPTION - A

Paper : CSC-HG-3016

(Programming in Python)

OPTION - B

Paper : CSC-HG-3026

(Computer Network and Internet Technologies)

OPTION - C

OPERATING SYSTEMS

Paper : CSC-HG-3036/CSC-RC-3016/TCA-HG-RC-3016

OPTION - D

ICT HARDWARE

Paper : CSC-HG-3046/TIT-RC-3016

Full Marks : 60

Time : Three hours

***The figures in the margin indicate
full marks for the questions.***

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