3 (Sem–3/CBCS) MAT HC 3

2023

MATHEMATICS

(Honours Core)

Paper : MAT–HC–3036

(Analytical Geometry)

Full Marks : 80

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer ***all*** the questions : 1×10=10

(a) When the origin is shifted to a point on the x-axis without changing the direction of the axes, the equation of the line $2x+3y-6=0$ takes the form $lx+my=0$. What is the new origin ?

(b) Find the centre of the ellipse $2x^2+3y^2-4x+5y+4=0$.

(c) Find the angle between the lines represented by the equation
$x^2 + xy - 6y^2 = 0$.

(d) Transform the equation $\frac{1}{r} = 1 + \cos\theta$ into cartesian form.

(e) Find the equation of the tangent to the conic $y^2 - xy - 2x^2 - 5y + x - 6 = 0$ at the point $(1, -1)$.

(f) Express the non-symmetric form of equation of a line $\frac{y}{p} + \frac{z}{c} = 1,\ x = 0$ in symmetric form.

(g) Write down the standard form of equation of a system of coaxial spheres.

(h) Write down the equation of a cone whose vertex is origin and the guiding curve is $ax^2 + by^2 + cz^2 = 1$,
$lx + my + nz = p$.

(i) Define a right circular cylinder.

(j) Find the equation of the tangent plane to the conicoid

$$ax^2 + by^2 + cz^2 = 1$$

at the point (α, β, γ) on it.

2. Answer **all** the questions : 2×5=10

(a) If $\left(at^2, 2at\right)$ is the one end of a focal chord of the parabola $y^2 = 4ax$, find the other end.

(b) Show that the equation of the lines through the origin, each of which makes an angle α to the line $y = x$ is

$$x^2 - 2xy\sec 2\alpha + y^2 = 0.$$

(c) Find the point where the line

$$\frac{x-1}{2} = \frac{y-2}{-3} = \frac{z+3}{4}$$

meets the plane $x + y + z = 3$.

(d) Find the equation of the sphere passing the points

$(0, 0, 0)$, $(a, 0, 0)$, $(0, b, 0)$, $(0, 0, c)$

(e) Find the equation of the plane which cuts the surface

$2x^2 - 3y^2 + 5z^2 = 1$ in a conic whose centre is $(1, 2, 3)$.

3. Answer ***any four*** questions : 5×4=20

(a) Show that the equation of the tangent to the conic $\frac{l}{r} = 1 + e\cos\theta$ at the point whose vertical angle is α is given by

$$\frac{l}{r} = e\cos\theta + \cos(\theta - \alpha).$$

(b) Prove that the line $lx + my = n$ is a normal to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, if

$$\frac{a^2}{l^2} + \frac{b^2}{m^2} = \frac{(a^2 - b^2)}{n^2}.$$

(c) Find the asymptotes of the hyperbola $2x^2 - 3xy - 2y^2 + 3x + y + 8 = 0$ and derive the equations of the principal axes.

(d) Prove that the lines

$$\frac{x+5}{3}=\frac{y+4}{1}=\frac{z-7}{-2} \text{ and}$$

$3x+2y+z-2=0=x-3y+2z-13$ are coplanar. Find the equation of the plane in which they lie.

(e) The section of a cone whose guiding curve is the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1, z=0$ by the plane $x=0$, is a rectangular hyperbola. Prove that the locus of the vertex is $\frac{x^2}{a^2}+\frac{y^2+z^2}{b^2}=1$.

(f) Find the centre and the radius of the circle

$$x^2+y^2+z^2-8x+4y+8z-45=0,$$
$$x-2y+2z=3.$$

Answer **either** (a) **or** (b) from the following questions : 10×4=40

4. (a) (i) Find the point of intersection of the lines represented by the equation

$$ax^2+2hxy+by^2+2gx+2fy+c=0$$

(ii) Find the equation of the polar of the point (2, 3) with respect to the conic $x^2 + 3xy + 4y^2 - 5x + 3 = 0$.

5+5=10

(b) (i) Prove that the straight line $y = mx + c$ touches the parabola

$$y^2 = 4a(x + a) \text{ if } c = ma + \frac{a}{m}.$$

(ii) Find the asymptotes of the hyperbola $xy + ax + by = 0$.

5+5=10

5. (a) Discuss the nature of the conic represented by

$$9x^2 - 24xy + 16y^2 - 18x - 101y + 19 = 0$$

and reduce it to canonical form.

(b) (i) Prove that the sum of the reciprocals of two perpendicular focal chords of a conic is constant.

(ii) Show that the semi-latus rectum of a conic is the harmonic mean between the segments of a focal chord.

5+5=10

6. *(a)* *(i)* A variable plane makes intercepts on the co-ordinate axes, the sum of whose squares is a constant and is equal to k^2. Prove that the locus of the foot of the perpendicular from the origin to the plane is

$$\left(x^2+y^2+z^2\right)^2\left(x^{-2}+y^{-2}+z^{-2}\right)=k^2$$

(ii) Two spheres of radii r_1 and r_2 intersect orthogonally. Prove that the radius of the common circle is

$$\frac{r_1 r_2}{\sqrt{r_1^2+r_2^2}}.$$

5+5=10

(b) Show that the shortest distance between any two opposite edges of the tetrahedron formed by the planes $y+z=0$, $z+x=0$, $x+y=0$, $x+y+z=a$ is $\frac{2a}{\sqrt{6}}$ and that the three lines of shortest distance intersect at the point $x=y=z=-a$.

7. (a) (i) Define reciprocal cone. Show that the cones $ax^2 + by^2 + cz^2 = 0$ and $\frac{x^2}{a} + \frac{y^2}{b} + \frac{z^2}{c} = 0$ are reciprocal.

(ii) Find the equation of the right circular cylinder whose guiding curve is $x^2 + y^2 + z^2 = 9$, $x - y + z = 3$.

5+5=10

(b) (i) Find the equation of the director sphere to the ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1.$$

(ii) Show that from any point six normals can be drawn to a conicoid $ax^2 + by^2 + cz^2 = 1$.

5+5=10
