3 (Sem–4/CBCS) PHY HC 1

2023

PHYSICS

(Honours Core)

Paper : PHY–HC–4016

(Mathematical Physics–III)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Give short answers to the following questions : 1×7=7

   (a) Find the principal value of i^i.

   (b) Define a multiply connected region in complex plane.

   (c) Find the value of $L^{-1}\left\{\frac{1}{s(s-a)}\right\}$ for $s > a$.

(d) What does the equation $|z-i|=2$ represent ?

(e) State convolution theorem of Fourier transform.

(f) Write the transformation rule for a covariant tensor of rank two.

(g) Plot the number $e^{\left(1+i\frac{\pi}{4}\right)}$.

2. Answer the following questions : 2×4=8

(a) Define simple pole and essential singularity.

(b) Establish the shifting property of Fourier transform.

(c) Find inverse Laplace transform of $t^{-\frac{1}{2}}$.

(d) Show that the number of independent components of a skew-symmetric tensor of rank 2 in n-dimensional space is $\frac{n(n-1)}{2}$.

3. Answer ***any three*** questions of the following : 5×3=15

(a) Check the analyticity and hence find derivative of the function $f(z)=\sin z$. 3+2=5

(b) Find the value of the integral $\int_0^{1+i}\left(x-y-ix^2\right)dz$ along real axis from $z=0$ to $z=1$ and then along the line parallel to imaginary axis from $z=1$ to $z=1+i$.

(c) Find the Fourier sine transform of a function defined by

$$f(t)=\begin{cases}\cos \propto t, 0 \le t \le \dfrac{\pi}{\propto} \\ 0\ , \quad t > \dfrac{\pi}{\propto}\end{cases}$$

(d) Evaluate :

$$L^{-1}\left\{\frac{(s+1)}{s^2(s+2)^3}\right\}$$

(e) Define Levi-Civita symbol in three dimensional space. Show that

$(\vec{A}\times\vec{B})_i = \varepsilon_{ijk}\, A_j\, B_k$ 2+3=5

4. Answer the following questions : ***(any three)*** 10×3=30

(a) (i) Prove Cauchy-Riemann conditions for analytical functions. What is the sufficient condition for a function to be analytic ? 4+1=5

(ii) Show that $|z_1+z_2| \le |z_1|+|z_2|$ 3

(iii) Give Laurent series expansion for function $f(z)$. 2

(b) (i) What are symmetric and antisymmetric tensors ? Show that every tensor can be expressed as the sum of two tensors, one of which is symmetric and the other antisymmetric in a pair of co-varient or contravarient indices. 2+3=5

(ii) What is Kronecker delta ? Prove that Kronecker delta is a mixed tensor of second rank. 1+4=5

(c) (i) Define Laplace transform of a function $F(t)$. Show that

$L(1)=\dfrac{1}{s}$, $s>0$ and

$L(e^{kt})=\dfrac{1}{s-k}$, $s>k$ 1+2+2=5

(ii) Find the inverse Laplace transform of

$$\frac{6}{2s-3}-\frac{3+4s}{9s^2-16}+\frac{8-6s}{16s^2+9}$$ 5

(d) Find the Fourier transform of

$$f(x)=\begin{cases}1-x^2, & |x|\le 1\\ 0 & |x|>1\end{cases}$$

Hence, evaluate :

$$\int_0^{\infty}\frac{x\cos x-\sin x}{x^3}\cos\frac{x}{2}dx$$

(e) Evaluate *any two* of the following integrals : 5×2=10

(i) $$\int_{-\infty}^{+\infty}\frac{\sin x}{x}\,dx$$

(ii) $$\int_0^{\infty}\frac{dx}{x^2+1}$$

(iii) $$\int_0^{2\pi}\frac{d\theta}{5+4\cos\theta}$$

(f) (i) The Laplace transform of $\sin 3t=\frac{3}{s^2+9}$ and the Laplace transform of $\cos 5t=\frac{s}{s^2+25}$. Find the Laplace transform of $5\sin 3t+3\cos 5t$ using linearity property of Laplace transform. 5

(ii) Find the inverse Laplace transform of $4s+5/(s-1)^2(s-2)$. 5