

Total number of printed pages-7

3 (Sem-4/CBCS) STA HC 2

2023

STATISTICS

(Honours Core)

Paper : STA-HC-4026

(Linear Models)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions as directed :

1×7=7

(a) In regression analysis, the variable that is being predicted is

(i) the independent variable

(ii) the dependent variable

(iii) usually denoted by x

(iv) usually denoted by r

(Choose the correct option)

Contd.

(b) The coefficient of determination is

- (i) equal to zero
 - (ii) the ratio of explained and total variation
 - (iii) usually less than zero
 - (iv) 100% of $(1 - r^2)$
- (Choose the correct option)

(c) In least square estimation, which of the following is not a required assumption about the error term ?

- (i) The expected value of the error term is one
- (ii) The variance of the error term is the same for all values of x
- (iii) The values of the error term are independent
- (iv) The error term is normally distributed

(Choose the correct option)

(d) If the regression equation is equal to $Y = 23.6 - 54.2X$, then 23.6 is the _____ while -54.2 is the _____ of the regression line.

- (i) slope, intercept
- (ii) slope, regression coefficient
- (iii) intercept, slope
- (iv) radius, intercept

(Choose the correct option)

(e) Analysis of variance is a statistical method of comparing the _____ of several populations.

- (i) standard deviations
- (ii) variances
- (iii) means
- (iv) None of the above

(Choose the correct option)

(f) The sum of squares due to _____ measures the variability of the observed values around their respective treatment means

- (i) treatment
- (ii) error
- (iii) interaction
- (iv) total

(Choose the correct option)

(g) All OLS estimators are linear estimators.
(Write True or False)

2. Answer the following questions briefly :

2×4=8

- (a) State some applications of the analysis of variance.
- (b) What do you understand by components of variation ?
- (c) Define estimability of linear parametric functions.
- (d) Define R^2 in the context of a linear model.

3. Answer **any three** of the following questions :

5×3=15

(a) What is a linear model ? Discuss different types of linear models.

(b) A sample of 20 observations on X and Y gave the following data :

$$\begin{aligned}\sum Y &= 21.9 & \sum (Y - \bar{Y})^2 &= 86.9 \\ \sum X &= 186.2 & \sum (X - \bar{X}) &= 215.4 \\ & & \sum (X - \bar{X})(Y - \bar{Y}) &= 106.4\end{aligned}$$

Estimate the regression equation of Y on X and X on Y .

(c) Consider the one-way AOV model

$$y_{ij} = \mu + \alpha_i + \varepsilon_{ij}, \text{ for } i = 1, 2 \text{ and } j = 1, 2, 3$$

Examine if μ, α_1, α_2 are estimable without any constraints.

(d) In what respects do AOV, regression analysis and AOCOV differ ? Discuss briefly.

(e) Write a note on the technique of hypothesis testing in case of simple regression models.

4. Answer **either (a) or (b)**: 10

(a) State and prove the Gauss-Markov theorem.

(b) What is analysis of variance (AOV)? What are the basic assumptions associated with it? What are the remedies, if the assumptions are violated?

5. Answer **either (a) or (b)**: 10

(a) Define a linear regression model. Write the basic assumptions of the linear model. Estimate the parameters of the model.

(b) Give linear model (fixed effect) for two-way classification (one observation per cell) and state its assumptions. Derive the analysis of variance of two-way classification through the method of least squares.

6. Answer **either (a) or (b)**: 10

(a) Using the following data

Y: 65 57 57 54 66

X: 26 13 16 -7 27

estimate the regression line $Y = \alpha + \beta X$, test the hypothesis that $\beta = 0$ against the alternative $\beta < 0$ at 5% level of significance, also construct 95% confidence interval for β .

(Given $t_{0.05,3} = 2.353$)

(b) Derive the 'analysis of covariance' for a one-way layout (with one consistent variable only).