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3 (Sem-6/CBCS) PHY HE 3

2023

PHYSICS

(Honours Elective)

Paper : PHY-HE-6036

(Advanced Mathematical Physics-II)

Full Marks : 80

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer **any ten** of the following :

1×10=10

- (a) State Hamilton's Principle.
- (b) Define Generalized co-ordinates.
- (c) What is meant by a Geodesic?

Contd.

(d) Generalized coordinates are:

- (i) Independent
- (ii) Dependent
- (iii) Spherical polar coordinates
- (iv) None of the above

(e) A particle is moving doing the arc of a circle. The suitable generalized coordinate (s) is/are :

- (i) r only
- (ii) θ only
- (iii) r and θ both
- (iv) x and y

(f) For a conservative system, the potential energy does not depend upon :

- (i) force
- (ii) generalized velocity
- (iii) generalized coordinator
- (iv) All of the above

(g) For a conservative system Hamiltonian represents :

(i) $T + V$

(ii) $T - V$

(iii) $L + V$

(iv) $2T - V$

(h) If the Lagrangian of a system does not depend on time explicitly, then :

(i) the Hamiltonian can not be constant

(ii) the Hamiltonian is constant

(iii) potential energy is constant

(iv) kinetic energy is constant

(i) The shortest distance between two points in a plane is :

(i) an arc of a circle

(ii) an arc of ellipse

(iii) a straight line

(iv) arbitrary arc

- (j) What is Mutually Exclusive Events ?
- (k) Give an example of Collectively Exhaustive events.
- (l) What is Probability Distribution Function ?
- (m) What is Compound Probability ?
- (n) Define Cyclic group.
2. Answer **any five** of the following : $2 \times 5 = 10$
- (a) Write Euler - Lagrange's differential equations.
- (b) Explain Principle of Least action.
- (c) What do you mean by canonical coordinates ?
- (d) What do you mean by *Lagrangian* and *Hamiltonian* ?
- (e) Define Mathematical Probability and Statistical Probability.
- (f) Define *relation* and *mapping*.

- (g) What are Homomorphism and Isomorphism of group ?
- (h) If $A = \{4, 5\}$ and $B = \{1, 2, 3\}$ are two sets, then find their Cartesian products $A \times B$ and $B \times A$.
3. Answer **any four** of the following : $5 \times 4 = 20$
- (a) Find the equation of a motion of a simple pendulum using Lagrange's equation.
- (b) What are Poisson - Brackets and Lagrange - Brackets ? Prove that both of them do not obey the commutative law of algebra.
- (c) Define a group and state its postulates.
- (d) Show that in a group G , identity elements e is unique.
- (e) Show that a non empty subset H of a group G is a subgroup of G iff
- (i) $a, b \in H \Rightarrow ab \in H$
- (ii) $a \in H \Rightarrow a^{-1} \in H$

(f) Prove that if $P(A)$ and $P(B)$ are probabilities of two Mutually Exclusive Events A and B then the probability of either of them is $P(A \text{ or } B) = P(A) + P(B)$.

4. Answer **any four** of the following : $10 \times 4 = 40$

(a) Using Euler-Lagrange equation prove that the shortest distance between two fixed points in a plane is a straight line.

$3 + 7 = 10$

(b) Use Hamilton's Principle to find the equation of motion of one dimensional harmonic oscillator. What is modified Hamilton's Principle ?

$7 + 3 = 10$

(c) Show that a non empty subset H of a group G is a subgroup of G iff

(i) $a, b \in H \Rightarrow ab \in H$

(ii) $a \in H \Rightarrow a^{-1} \in H$

$5 + 5 = 10$

(d) Show that under canonical transformation, Poisson-Bracket is invariant.

(e) What is the law of Conditional Probabilities ? State and prove Bayes' theorem.

$4 + 6 = 10$

(f) What is Random Variable ? Explain about probability distribution of a discrete random variable and a continuous random variable.

$2 + 4 + 4 = 10$

(g) What is Normal Distribution ? Explain with schematic diagrams that how the shape of Normal Distribution curves depends upon the Mean and Standard Deviation. Write importance of Normal Distribution.

$2 + 6 + 2 = 10$

(h) Explain the Poisson distribution. Consider an emergency room of a hospital where the past records indicate an average of 5 arrivals daily. The demand for emergency room service at this hospital is distributed according to a Poisson distribution. Calculate the probability of exactly 0, 1, 2, 3, 4 and 5 arrivals. What is the probability of more than 3 arrivals ?

$3 + 5 + 2 = 10$