

Total number of printed pages-7

3 (Sem-3/CBCS) STA HC 1

2023

**STATISTICS**

(Honours Core)

Paper : STA-HC-3016

**( Sampling Distributions )**

Full Marks : 60

Time : Three hours

**The figures in the margin indicate full marks for the questions.**

1. Answer the following questions as directed :

1×7=7

(a) The standard error of the sampling distribution of the statistic ( $\bar{x}$ ) is

(Choose the correct option)

(i)  $\sigma^2 \sqrt{2/n}$

(ii)  $\sigma/\sqrt{n}$

(iii)  $\sqrt{\sigma^2/2n}$

(iv) None of the above

Contd.

(b) The cumulative distribution function of the largest order statistic  $X_{(n)}$  is given by \_\_\_\_\_ . (Fill in the blank)

(c) For large  $n$  if  $X \sim N(nP, nPQ)$ , then

$$Z = \frac{X - nP}{\sqrt{nPQ}} \text{ follows}$$

(Choose the correct option)

(i)  $N(0, \sigma^2)$

(ii)  $N(\mu, \sigma^2)$

(iii)  $N(0, 1)$

(iv)  $N(\mu, \sigma)$

(d) If  $X_i, i = 1, 2, \dots, n$  are  $n$  independent normal variates with mean  $(\mu_i)$  and SD

$(\sigma_i)$ , then  $\sum_{i=1}^n \left( \frac{X_i - \mu}{\sigma} \right)^2$  is a chi-square

variate with  $n$  d.f.

(Write true or false)

(e) The probability of type I error is called \_\_\_\_\_ . (Fill in the blank)

(f) State the pdf of Fisher's  $t$ -distribution.

(g) Equality of two population variances can be tested by

(Choose the correct option)

(i)  $t$ -test

(ii)  $F$ -test

(iii) Both (i) and (ii)

(iv) None of the above

2. Answer the following questions :  $2 \times 4 = 8$

(a) Derive the cumulative distribution function of  $X_{(1)}$ .

(b) Write any two applications of chi-square statistic.

(c) Explain one tailed and two tailed tests.

(d) Write the assumptions for students  $t$ -test.

3. Answer **any three** questions from the following :  $5 \times 3 = 15$

(a) Explain in brief the test used for testing the difference between two proportions for large samples.

(b) Find the joint distribution of  $r^{\text{th}}$  and  $s^{\text{th}}$  order statistics ( $r < s$ ) in taking random sample from a continuous distribution.

(c) Derive cumulant generating function (c.g.f.) of chi-square distribution. Also find its mean and variance using c.g.f.

(d) Define  $F$  statistic. Write down the p.d.f. of Snedecor's  $F$  distribution. Derive the mode of  $F$  distribution.  $1+1+3=5$

(e) In  $F(n_1, n_2)$  distribution and if  $n_2 \rightarrow \infty$ , then prove that  $n_1 F$  follows chi-square distribution with  $n_1$  d.f.

Answer **either 4. (a) or 4. (b)** :

4. (a) Obtain the distribution of sample median in case of order statistics. 10

(b) (i) Let  $X_1$  and  $X_2$  be two independent normal variates with the same normal distribution  $N(\mu, \sigma^2)$ . Obtain the distribution of

$$Y = \frac{X_1 + X_2 - 2\mu}{\sqrt{|X_1 - X_2|^2}} \quad 5$$

(ii) If  $X$  is  $t$ -distributed with  $K$  degrees of freedom, show that  $\frac{1}{1+(X^2/K)}$  has a beta distribution. 5

Answer either 5. (a) or 5. (b) :

5. (a) If  $X_1$  and  $X_2$  are two independent chi-square variate with  $n_1$  and  $n_2$  d.f. respectively, then show that  $X_1/X_2$  is a  $\beta_2(n_1/2, n_2/2)$  variate. 10

(b) (i) Describe the steps in detail for testing a statistical hypothesis. 5

(ii) For  $t$ -distribution with  $n$  d.f., derive the mean deviation about mean. 5

Answer either 6. (a) or 6. (b) :

6. (a) (i) Derive the probability density function of student's  $t$ . 7

(ii) Comment on the graph of  $t$ -distribution. 3

(b) Write three applications of  $F$  distribution.

Let  $X_1$  and  $X_2$  be a random sample of size 2 from  $N(0, 1)$  and  $Y_1$  and  $Y_2$  be a random sample of size 2 from  $N(1, 1)$  and let  $Y_i$ 's be independent of  $X_i$ 's. Find the distribution of the following :

3+7=10

(i) 
$$\frac{(X_1 + X_2)^2}{(X_2 - X_1)^2}$$

(ii) 
$$\frac{(Y_1 + Y_2 - 2)^2}{(X_2 - X_1)^2}$$