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3 (Sem-5/CBCS) PHY HC 1

2023

PHYSICS

(Honours Core)

Paper : PHY-HC-5016

(Quantum Mechanics and Applications)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions : $1 \times 7 = 7$

- (a) Why eigenvalues and eigenfunctions of Hermite operators are very important for a quantum physicist?
- (b) Stationary states are those for which the probability density ρ is
- (i) time-dependent
 - (ii) time-independent
 - (iii) space-dependent
 - (iv) space-independent

Contd.

- (c) Why $\psi = x^n$ is not an acceptable wave function ?
- (d) Can kinetic energy and linear momentum of a quantum system be specified simultaneously ? Give reason.
- (e) Write down the Schrödinger equation for a particle in a square well potential of infinite depth.
- (f) The z-component of spin angular momentum can take values :
- (i) $\pm h$
- (ii) $\pm \frac{\hbar}{2}$
- (iii) $\pm \hbar$
- (iv) $\pm 2\hbar$
- (g) An atomic state is denoted by ${}^4D_{5/2}$. What should be the minimum number of electrons involved for this state ?

2. Answer the following questions : $2 \times 4 = 8$

(a) Show that $[\hat{L}_x, \hat{L}_y] = i\hbar \hat{L}_z$.

- (b) How much energy is released, when a $3d$ electron in the hydrogen atom makes a transition to a $2p$ state?
- (c) Calculate Lande's g factor for a p -electron.
- (d) Show that in a non-dispersive medium, the group velocity is equal to the phase velocity.

3. Answer **any three** of the following questions : 5×3=15

(a) What do you mean by dynamical variables and expectation value of a dynamical variable? Obtain quantum mechanical operators corresponding to linear momentum, kinetic energy and Hamiltonian of a system. 1+1+3=5

(b) A free particle is initially localized in the range $-a < x < a$ as :

$$\psi(x, 0) = u(x) = A, \quad -a < x < 0$$

$$u(x) = 0, \quad \text{otherwise}$$

where A , a are real and positive. Find $\psi(x, t)$.

(c) Starting from time independent Schrödinger equation in polar coordinate for hydrogen atom, show that for azimuthal angle, the probability density of electron is constant. What is its significance?

(d) A particle in the ground state is located in one dimensional potential well of width L with absolutely impenetrable walls $0 < x < L$. Find probability of finding the particle in the region

$$\frac{L}{3} < x < \frac{2L}{3}.$$

(e) What are identical particles? Show that when two identical particles try to occupy same quantum state, then anti-symmetric wave function becomes zero. Why Pauli's exclusion principle is not valid for Bosons? 1+3+1=5

4. Answer **any three** of the following questions : 10×3=30

(a) (i) Explain the meaning of probability current density for a quantum system. Deduce an expression for the probability current density for three dimensional motion and the law of conservation of probability density. 1+3+1=5

- (ii) The wave function of a particle moving in one dimension is given to be

$$\psi(x) = \begin{cases} \sqrt{\frac{15}{a}} A (a^2 - x^2) & \text{for } -a \leq x \leq 0 \\ 0 & \text{for } |x| > a \end{cases}$$

Find the value of A that will normalise $\psi(x)$ and calculate the expectation values of x and p .

1+2+2=5

[where the notations have their usual meaning.]

- (b) (i) For a linear harmonic oscillator, obtain the ground state wave function. Make a plot of the first and second energy eigenfunctions.

7+1=8

- (ii) Compare the ground state classical and quantum mechanical probability of the oscillator. What happens when the quantum numbers become very large?

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- (c) From the polar equation of hydrogen atom separate the radial part and using Frobenius method find the energy states.

3+7=10

(d) (i) What is Zeeman effect? Give the explanation of normal Zeeman effect on the basis of classical theory and obtain an expression for Zeeman shift. $1+5=6$

(ii) Explain why normal Zeeman effect occurs only in atoms with even number of electrons. 2

(iii) An spectrometer can resolve spectral lines separated by 0.03nm . How much magnetic field will have to be applied to a source of 422.7nm line, so that the triplet is just resolved in normal Zeeman effect? 2

(e) Differentiate between L-S and J-J coupling schemes.

The wavelengths of D lines of sodium are 5896\AA and 5890\AA . Calculate the (a) energy of the levels from which these spectral lines originate (b) separation in eV between the two p -levels in sodium atom. Given that the ionisation energy of sodium is equal to $5.13eV$.

$5+5=10$

(f) Describe Stern-Gerlach experiment with a suitable diagram and explain on the basis of quantum theory.

In a Stern-Gerlach experiment silver atoms traverse a distance 0.1m in a non-homogeneous field of gradient $55\text{ Tesla } m^{-1}$. If the velocity of silver atom is 450ms^{-1} , calculate the separation between the two traces on the collector plate.

[Bohr magneton = $9.27 \times 10^{-24}\text{JT}^{-1}$,
Mass of silver atom = $1.79 \times 10^{-25}\text{kg}$]

3+5+2=10
