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3 (Sem-5/CBCS) PHY HE 3

2023

**PHYSICS**

(Honours Elective)

Paper : PHY-HE-5036

**(Advanced Mathematical Physics - I)**

Full Marks : 60

Time : Three hours

**The figures in the margin indicate full marks for the questions.**

1. Answer the following questions :  $1 \times 7 = 7$

(a) What is isomorphism in case of a vector space ?

(b) Define associated tensor.

(c) What is field ? Give two examples.

(d) State quotient law of tensors.

(e) Write the scalar triple product  $\bar{A} \cdot (\bar{B} \times \bar{C})$  using tensor notation.

Contd.

(f) What is Moment of Intertia tensor ?

(g) If  $A = \begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix}$ , find  $2^A$ .

2. Answer the following : 2×4=8

(a) Show that diagonalizing matrix of a symmetric matrix is orthogonal.

(b) Show that the vectors  $W_1 = [2, 1, 1]$ ,  
 $W_2 = [-2, 1, 2]$  and  $W_3 = [0, 0, 1]$   
are linearly independent.

(c) What is Minkowski space ? Define a four vector in this space.

(d) Verify Cayley-Hamilton theorem for

the matrix  $A = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$ .

3. Answer **any three** of the following question ; 5×3=15

(a) What is binary operation ? Determine the identity element and inverse for the following binary operation :

$(a, b) * (c, d) = (ac, bc + d)$ . 1+4=5

(b) Diagonalize the matrix

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$$

(c) (i) If a contravariant tensor of rank two is symmetric in one co-ordinate system, show that it is symmetric in any co-ordinate system. 3

(ii) If  $A_\lambda$  is a covariant tensor of rank one, verify whether  $\frac{\partial A_\lambda}{\partial x^\mu}$  is a tensor or not. 2

(d) (i) Find the number of independent components of a second rank symmetric tensor in  $n$ -dimensional space. 2

(ii) Using the relation

$$ds^2 = g_{ij} dx^i dx^j, \text{ prove that } g_{ij}$$

is a symmetric tensor. 3

(e) Using tensor-analysis, show that :

$$2+3=5$$

(i)  $\epsilon_{ils} \epsilon_{mls} = 2\delta_{im}$

(ii)  $\bar{\nabla} \cdot \bar{A}$  is an invariant.

4. Answer **any three** of the following questions :  $10 \times 3 = 30$

~~(a)~~ (i) Define basis and dimension of a linear vector space. If  $x, y, z$  are linearly independent vectors, determine whether the vector  $x+y, y+z$  and  $z+x$  are linearly dependent or not.  $2+3=5$

(ii) Use  $\epsilon_{ijk}$  to find the vector associated with the following anti-symmetric tensor of rank two :

$$\begin{bmatrix} 0 & 1 & -3 \\ -1 & 0 & 2 \\ 3 & -2 & 0 \end{bmatrix}$$

and to express cross product of vectors  $\bar{A}$  and  $\bar{B}$ .  $3+2=5$

(b) (i) What is Group ? Check whether the set  $I$  of all integers with the binary operation  $*$  defined by  $a * b = a + b + 1$  forms a Group.

1+4=5

(ii) Show that every linearly independent vector belonging to a vector space has a unique representation as a linear combination of its bases vector.

5

(c) (i) Using tensor analysis prove the following vector identities :

2+2+3=7

(a)  $\vec{\nabla} \cdot (\vec{\nabla} \times \vec{A}) = 0$

(b)  $\vec{\nabla} \times (\phi \vec{A}) = \phi (\vec{\nabla} \times \vec{A}) + \vec{\nabla} \phi \times \vec{A}$

(c)  $\vec{\nabla} \times (\vec{\nabla} \times \vec{A}) = \vec{\nabla} (\vec{\nabla} \cdot \vec{A}) - \nabla^2 \vec{A}$

(ii) Find the second order antisymmetric tensor associated

with the vector  $2\hat{i} - 3\hat{j} + \hat{k}$ . 3

(d) (i) Solve the coupled linear differential equations using matrix method :

$$y_1' = 2y_1 + 3y_2$$

$$y_2' = 4y_1 + y_2$$

where  $y_1(0) = 2, y_2(0) = 1$ . 5

(ii) Show that in Cartesian co-ordinate system, the contravariant and covariant components of a vector are identical. 5

(e) What is matrix tensor  $g_{qr}$  ? Calculate the co-efficients of matrix tensor in spherical polar co-ordinate and then write the matrix tensor. Prove that the matrix tensor  $g_{qr}$  is a symmetric covariant tensor of order 2. 2+2+6=10

(f) (i) State Hooke's law in elasticity using tensor notation. If  $\epsilon_{ij}$ 's denote fractional deformation, establish the relation,

$$\delta = \epsilon_{xx} + \epsilon_{yy} + \epsilon_{zz}, \text{ where } \delta \text{ is the change in volume associated with the deformation.} \quad 2+5=7$$

(ii) Prove that eigenvalues of a hermitian matrix are real. 3

$a_2 = a_1$   
 $\text{cent} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} + \text{Det}$   
 $(a \times b) \rightarrow (b \times a) = (b \times a) + (a \times b)$   
 $b_1 = -\frac{3}{4}$