(e) X is a binomial variate with parameters n and p and f_{v_1,v_2} is an F-statistic with v_1 and v_2 df. Prove that

$$P(X \le k-1) = P\left[f_{2k}, 2(n-k+1) > \left(\frac{n-k+1}{k}\right)\left(\frac{p}{1-p}\right)\right]$$

(f) If X_1 and X_2 are independent χ^2-variates with n_1 and n_2 df respectively and $U = X_1/(X_1+X_2)$ and $V = X_1 + X_2$ are independently distributed, show that U is a $\beta_1\left(\frac{n_1}{2}, \frac{n_2}{2}\right)$ variate and V is a χ^2 variate with (n_1+n_2) df.

(g) Express the constants y_0, a and m of the distribution

$$dF(x) = y_0\left(1 - x^2/a^2\right)^m dx, -a < x < a$$

in terms of its μ_2 and β_2.

(h) If the random variables X_1 and X_2 are independent and follow chi-square distribution with n df, show that $\sqrt{n}(X_1 - X_2)/2\sqrt{X_1 X_2}$ is distributed as student's t with df independently of $(X_1 + X_2)$.

Total number of printed pages–8

3 (Sem–3/CBCS) STA HC 1

2022

STATISTICS

(Honours)

Paper : STA–HC–3016

(Sampling Distributions)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer ***any seven*** of the following questions : 1×7=7

   (a) If X and Y are *two* independent chi square variates with n_1 and n_2 d.f. respectively, then $U = X/Y$ follows

   (i) $\beta_2(n_1/2, n_2/2)$

   (ii) $\beta_1(n_1, n_2)$

   (iii) F distribution

   (iv) None of the above

   (Choose the correct option)

(b) The skewness in a chi-square distribution will be zero, if

(i) $n \to \infty$

(ii) $n = 0$

(iii) $n = 1$

(iv) $n < 0$

(Choose the correct option)

(c) Level of significance is the probability of

(i) type I error

(ii) type II error

(iii) not committing error

(iv) Any of the above

(Choose the correct option)

(d) Student's *t*-test is applicable in case of

(i) small samples

(ii) samples of size between 5 and 30

(iii) large samples

(iv) None of the above

(Choose the correct option)

(e) Equality of *two* population variances can be tested by

(i) t-test

(ii) F-test

(iii) Both (i) and (ii)

(iv) Neither (i) nor (ii)

(Choose the correct option)

(f) Test of hypothesis $H_0 : \mu = 70$ vs $H_1 : \mu > 70$ leads to

(i) one sided left tailed test

(ii) one sided right tailed test

(iii) two tailed test

(iv) None of the above

(Choose the correct option)

(g) Degress of freedom for statistic – χ^2 in case of contingency table of order (2×2) is

(i) 3

(ii) 4

(iii) 2

(iv) 1

(Choose the correct option)

(h) Analysis of variance utilises

(i) F-test

(ii) χ^2-test

(iii) Z-test

(iv) t-test

(Choose the correct option)

(i) If β is the probability of type II error, the power of the test is ______. *(Fill in the blank)*

(j) Critical region is also known as ______. *(Fill in the blank)*

(k) t-distribution with 1 d. f. reduces to ______. *(Fill in the blank)*

(l) The value of chi square varies from ______ to ______. *(Fill in the blanks)*

2. Answer **any four** questions of the following : 2×4=8

(a) A random sample of size 4 is drawn from the discrete uniform distribution

$$P(X=x)=\frac{1}{6};\ x=1,2,3,4,5,6$$

Obtain the distribution of the smallest and largest order statistic.

(b) Obtain the moment generating function of chi square distribution.

(c) Define sampling distribution and standard error.

(d) State the applications of order statistics.

(e) Under what conditions is χ^2 test valid ?

(f) State the important applications of F distributions.

(g) Define critical region and level of significance.

(h) State the applications of t-distribution.

3. Answer ***any three*** of the following questions : 5×3=15

(a) Derive Fisher's t-distribution.

(b) For 2×2 contingency table

a	b
c	d

, prove that chi square test of independence given

$$\chi^2 = \frac{N(ad-bc)^2}{(a+c)(b+d)(a+b)(c+d)},\quad N=a+b+c+d.$$

(c) Show that the mgf of $y=\log\chi^2$, where χ^2 follows chi square distribution with n df is

$$M_Y(t)=2^t\,\Gamma\!\left(\frac{n}{2}+t\right)\Big/\Gamma(n/2)$$

(d) Show that for t-distribution with n df the mean deviation about mean is

$$\sqrt{n}\,\Gamma\left(\frac{n-1}{2}\right) \Big/ \Gamma\left(\frac{1}{2}\right)\Gamma\left(\frac{n}{2}\right)$$

(e) There are two populations and P_1 and P_2 are the proportions of members in the two populations belonging to 'low income' group. It is desired to test the hypothesis $H_0 : P_1 = P_2$. Explain clearly, the procedure that you would follow to carry out the above test at 5% level of significance.

(f) Let $X_1, X_2, ..., X_n$ be a random sample from $N(0,1)$. Define $\overline{X}_k = \frac{1}{k}\sum_{i=1}^{k} X_i$ and

$$\overline{X}_{n-k} = \frac{1}{n-k}\sum_{i=k+1}^{n} X_i$$

Find the distribution of

(a) $\frac{1}{2}\left(\overline{X}_k + \overline{X}_{n-k}\right)$

(b) $k\overline{X}_k^{\,2} + (n-k)\overline{X}_{n-k}^{\,2}$

(g) Show how probability points of $F(n_2, n_1)$ can be obtained from those of $F(n_1, n_2)$.

(h) For the exponential distribution $f(x) = e^{-x}, x \geq 0$; show that the c.d.f. of $X(n)$ in a random sample of size n is $F_n(X) = \left(1 - e^{-x}\right)^n$.

4. Answer ***any three*** of the following questions : 10×3=30

(a) If X_1 and X_2 are two independent χ^2-variates with n_1 and n_2 df respectively, then show that X_1/X_2 is a $\beta_2(n_1/2, n_2/2)$ variate.

(b) Find the variance of the t-distribution with n df $(n > 2)$.

(c) Derive the relation between F and χ^2 distribution.

(d) Derive Snedecor's F distribution.