2019

MATHEMATICS

(Major)

Paper : 5.5

(Probability)

Full Marks : 60

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Answer the following questions : 2×4=8

(a) For what type of events A and B—

(i) $P(A \cup B) = P(A) + P(B)$;

(ii) $P(A \cap B) = P(A)\,P(B)$? 1+1=2

(b) *(i)* Find c, so that the function

$$f(x) = \begin{cases} cx, & 0 < x < 3 \\ 0, & \text{otherwise} \end{cases}$$

is a density function.

(ii) Point out the error in the statement "the probability that a student will commit exactly one mistake is 0·5 and the probability that he will commit at least one mistake is 0·03". 1+1=2

(c) Define the following : 1+1=2

(i) Random variable (r.v.)

(ii) Mathematical expectation of an r.v.

(d) Write the condition on—

(i) n, the number of trials;

(ii) p, the probability of success;

so that the Poisson distribution can be obtained as a limiting case of binomial distribution. 1+1=2

2. Answer any *four* of the following questions : 3×4=12

(a) Find the probability of not getting a 7 or 10 total on either of two tosses of a pair of fair dice.

(b) Prove or disprove, the second moment about any point a is minimum when taken about the mean μ, i.e.,

$$E(X-a)^2 \geq E(X-\mu)^2$$

(c) A box contains 7 red and 4 white marbles. Two marbles are drawn successively from the box without replacement and it is noted that the 2nd one is white. What is the probability that the first one is also white?

(d) 8 coins are tossed at a time, 256 times. Find the expected frequencies of success (getting a head).

(e) Show that the expectation of a discrete random variable X whose probability function is given by

$$f(x) = \left(\frac{1}{2}\right)^x; \quad x = 1, 2, 3, \cdots$$

is 2.

(f) Prove that the mean of a binomially distributed random variable is $\mu = np$.

3. Answer any *two* of the following questions : 5×2=10

(a) State and prove Bayes' theorem.

(b) A bag contains 5 white and 3 black balls, another bag contains 4 white and 5 black balls. From any one of these bags, a single draw of 2 balls is made. Find the probability that one of them would be white and the other black ball.

(c) The probabilities of n independent events are $p_1, p_2, \cdots, p_n$. Find the expression for the probability that at least one of the events will happen.

4. Answer any *two* of the following questions : 5×2=10

(a) A random variable X has the density function

$$f(x) = \frac{c}{x^2+1}$$

where $-\infty < x < \infty$.

(i) Find the value of c.

(ii) Find the probability that X^2 lies between $\frac{1}{3}$ and 1.

(b) The joint density function of two continuous random variables X and Y is

$$f(x, y) = \begin{cases} cxy, & 0 < x < 4,\ 1 < y < 5 \\ 0, & \text{otherwise} \end{cases}$$

(i) Find the value of c.

(ii) Find $P(X \geq 3, Y \leq 2)$.

(c) Two random variables X and Y are jointly distributed as follows :

$$f(x, y) = \frac{2}{\pi}(1 - x^2 - y^2);\ 0 < x^2 + y^2 < 1$$

Find the marginal distribution of X.

5. Answer any *two* of the following questions : 5×2=10

(a) Prove that the mathematical expectation of the product of two independent random variables is equal to product of their expectations.

(b) Prove that

$$\text{Var}(X) = E[\text{Var}(X|Y)] + \text{Var}[E(X|Y)]$$

(c) A random variable X has density function given by

$$f(x) = \begin{cases} 2e^{-2x}; & x \geq 0 \\ 0 \;\; ; & x < 0 \end{cases}$$

Find the—

(i) moment-generating function;

(ii) first four moments about the origin.

6. Answer any *two* of the following questions : 5×2=10

(a) In a binomial distribution, show that the variance is always less than the mean.

(b) What is meant by standard normal variate? Find the mean and variance of standard normal variate.

(c) Find the probability that in a sample of 10 tools chosen at random, exactly two will be defective by using the—

(i) binomial distribution;

(ii) Poisson approximation to the binomial distribution.

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